Analytical model for two-dimensional pollutant transport in defective GM/GCL/SL composite liners

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Abstract: This study presents an analytical model for two-dimensional pollutant transport in a three-layer composite liner system, comprising a geomembrane (GM), a geosynthetic clay liner (GCL), and a soil liner (SL), with a focus on the impact of defects in the GM. By utilizing Laplace and Fourier transforms, the model derives pollutant concentration distributions, incorporating processes such as convection, diffusion, adsorption, and degradation. Validation against COMSOL simulations demonstrated the model's accuracy. The findings reveal that traditional models significantly underestimate longitudinal pollutant migration and overestimate lateral migration. These insights emphasize the necessity for advanced analytical methods in order to enhance the design and effectiveness of landfill liner systems.

Keywords: Analytical model; Two-dimensional; Defective composite liner; Landfill; Geomembrane

1. Introduction

Landfills serve as a critical component of waste management systems, particularly for the disposal of municipal solid wastes and industrial by-products (Gómez-García et al., 2021; Ghosh et al., 2023; Ling et al. 2024; Nanda and Berruti, 2021; Qian et al., 2024; Woodman et al., 2017). However, one of the major concerns associated with landfills is the potential for leachate migration from the waste into the surrounding environment (Sobral et al., 2024; Wu et al., 2021; Zhang et al., 2021). To mitigate this, composite liner systems are widely used (Abiriga et al., 2020; Shu et al., 2019; Teng et al., 2021; Wijekoon et al., 2022). These systems typically consist of a geomembrane (GM), a geosynthetic clay liner (GCL), and a soil layer (SL), working in unison to contain leachate and prevent contaminants from seeping into groundwater and the surrounding soil (Trauger and Tewes, 2020). Despite the effectiveness of these composite liners under ideal conditions, defects in the liners can occur due to various factors such as construction issues, material degradation, or external stresses (Touze-Foltz et al., 2021; Rowe and Hamdan, 2022; Rowe et al., 2023; Sun et al., 2019). Such defects may compromise the integrity of the liners and result in pollutant leakage. Understanding the transport mechanisms of pollutants in these compromised systems is therefore crucial for assessing environmental risks and improving landfill designs.

The study of pollutant transport through landfill liners has been an area of considerable research over the past few decades. Numerous models have been developed to describe the transport of contaminants through liner systems. Traditionally, one-dimensional models have been used, focusing on vertical transport through the layers. For instance, Xie et al. (2013) developed a 1D model using Laplace transform to describe the diffusion of organic pollutants through a three-layer composite liner, providing an analytical solution as an alternative to numerical models. Similarly, Yu et al. (2018) proposed a migration and transformation model for pollutants in 1D layered porous media, comprehensively considering the effects of adsorption and biodegradation. Other researchers, such as Pu et al. (2019) and Feng et al. (2019b), developed models that considered the diffusion and transient migration of pollutants, respectively, further refining the predictions of pollutant behavior in composite liners.

Although these models have provided valuable insights into pollutant transport mechanisms, they often simplify the complex, multi-dimensional nature of real-world landfill scenarios. In reality, pollutant migration in defective liners is not confined to the vertical direction; it can also occur laterally, necessitating more sophisticated two-dimensional (2D) models. Former advancements have attempted to address this complexity. For instance, Dominijanni and Manassero (2021) provides analytical solutions for pollutant concentrations in the vertical and horizontal directions, aiding in the evaluation of the equivalence and effectiveness of composite liners. Rouholahnejad and Sadrnejad (2009) used 2D advection-diffusion-linear sorption with first order decay equation to assess leachate migration from the landfill to groundwater, the transport of pollutants after the leachate enters the surface was further clarified.

Despite the progress made in modeling pollutant transport through composite liners, several critical challenges remain. A major issue is the limited consideration of defects in the GM layer. These defects can drastically alter the containment efficacy of liner systems, leading to significant deviations from the predictions made by models that assume intact conditions. For instance, Xie et al. (2010) modeled the steady-state transport of pollutants through a defective GM and demonstrated that defects could substantially affect pollutant migration patterns, especially when varying GM conditions are considered. Moreover, current models often do not fully account for the coupled physical processes—such as diffusion, advection, retardation, and degradation—that occur within the liner system. These processes interact in complex and nonlinear ways, particularly in the presence of defects, making it challenging to accurately predict pollutant transport. The need for more precise、initial concentration distributions, as highlighted by Xie et al. (2014) and Sun et al. (2022), further complicates the modeling of defective systems. Additionally, there remains a significant gap in understanding how defects impact the transport of different types of pollutants, such as heavy metals and organic compounds, which behave differently within composite liners.

Given these challenges, this study aims to fill critical gaps in the understanding of pollutant transport in defective GM/GCL/SL composite liners. The primary objective is to present an innovative 2D analytical model that comprehensively examines convection, diffusion, adsorption, and degradation under defect conditions, supported by precise mathematical derivations and numerical validation. The findings are expected to enhance the effectiveness of containment strategies, ultimately leading to better protection of the environment from landfill-related pollution.

2. Mathematical model

2.1 Basic assumptions

As shown in Fig.1a, the composite liner system considered in this study comprises a GM, GCL, and SL (Fig.1b). The GM layer is assumed to be in a defective state, allowing direct contact with the leachate. In the context of the model, z_1 represents the thickness of the GM, z_2 represents the combined thickness of the GM and GCL, z_3 represents the combined thickness of the GM, GCL and SL. L_1 represents the width of the leak, and L_2 represents the length from the leak to the next leak in the GM. The model is based on the following assumptions: (1) The flow of leachate within the liner is steady-state and obeys Darcy's law; (2) Migration of metal pollutants through the non-defective GM is neglected; (3) Both the GCL and SL are assumed to be fully saturated and have uniform properties (Wu et al., 2015); (4) The effects of convection, diffusion, adsorption, and degradation are considered.

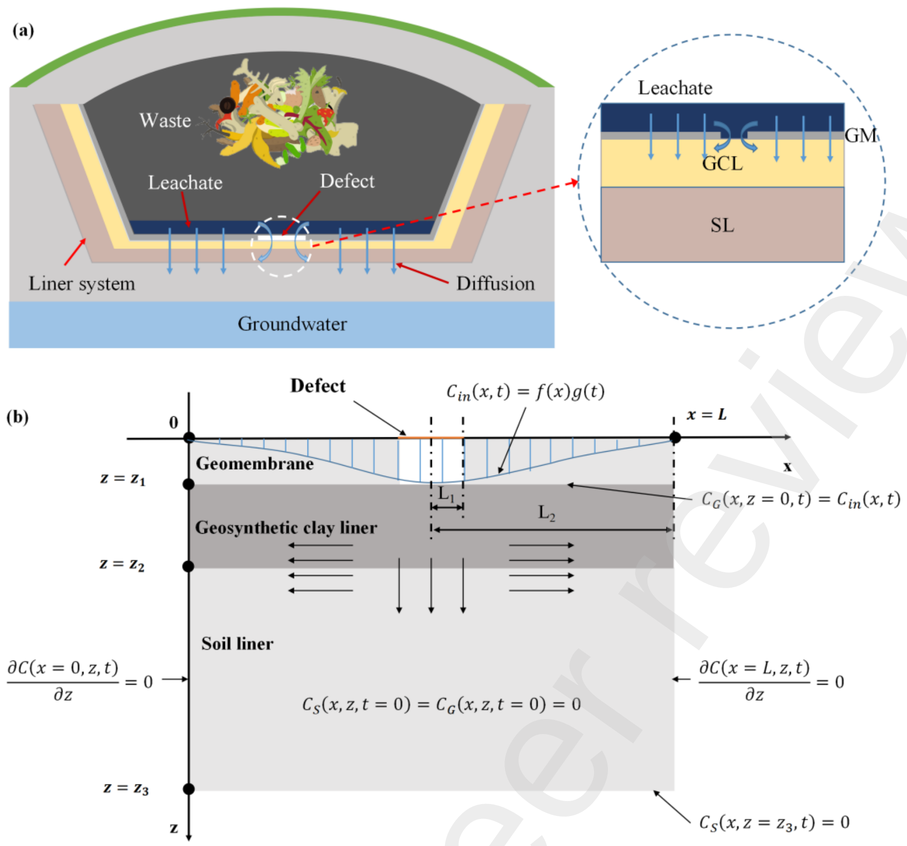

Fig. 1. The migration of leachate through the composite liner system:(a) schematic diagram; (b) mathematical model.

2.2 Governing equations and boundary conditions

Based on the above assumptions, the two-dimensional transport of pollutants in the GM/GCL/SL composite liner can be described by the equations of convection, diffusion, adsorption, and degradation .

For the GCL:

$$R_{d,G}\frac{\partial C_G}{\partial t} = D_{x,G}\frac{\partial^2 C_G}{\partial x^2} + D_{z,G}\frac{\partial^2 C_G}{\partial z^2} - v_G\frac{\partial C_G}{\partial z} - \lambda_G C_G \quad (1)$$

For the SL:

$$R_{d,S}\frac{\partial C_S}{\partial t} = D_{x,S}\frac{\partial^2 C_S}{\partial x^2} + D_{z,S}\frac{\partial^2 C_S}{\partial z^2} - v_S\frac{\partial C_S}{\partial z} - \lambda_S C_S \quad (2)$$

where C_i(i=G,S) represents the concentration of pollutants in the liner layer, which is a function of position and time; $R_{d,i}$ represents the adsorption retardation factor of the i-th layer of liner; $D_{x,i}$

and $D_{z,i}$ represent the diffusion coefficient in the x and z directions of the i-th layer, respectively; v_i is the convection coefficient in the liner layer; and λ_i represents the degradation constant of organic pollutants.

The expressions for the adsorption retardation factor (R_d) and degradation coefficient (λ) are respectively:

$$R_d = 1 + \frac{\rho K_d}{n} \tag{3}$$

$$\lambda = \frac{ln2}{t_{1/2}} \tag{4}$$

Where ρ is the density of the liner, K_d is the distribution coefficient of the liner and $t_{1/2}$ is the half-life of an organic pollutants.

Assuming the liner system has not been contaminated at the outset, the initial conditions of the liner system are :

$$C_S(x,z,t=0) = C_G(x,z,t=0) = 0 \tag{5}$$

$C_S(x,z,t)$ represents the concentration of SL, $C_G(x,z,t)$ represents the concentration of GCL.The boundary conditions for the entrance of the GM defect can be represented by a concentration function in terms of width (x) and time (t):

$$C_M(x,z=0,t) = C_{in}(x,t) \tag{6}$$

$C_M(x,z,t)$ represents the concentration of GM, the function $C_{in}(x,t)$ represents the concentration of the pollutant source, which is the product of a function $f(x)$ related to the width and a function $g(t)$ related to time.

$$C_{in}(x,t) = f(x)g(t) \tag{7}$$

The lower boundary of the composite liner is assumed to be a semi-infinite boundary.

$$C_S(x,z=z_3,t) = 0 \tag{8}$$

The left and right boundary condition of the model can be written as:

$$\frac{\partial C(x=0,z,t)}{\partial z}=0 \tag{9}$$

$$\frac{\partial C(x=L,z,t)}{\partial z}=0 \tag{10}$$

The concentration and flux at the interface between GCL and SL are equal, with expressions as follows:

$$C_G(x,z=z_1,t)=C_S(x,z=z_1,t) \tag{11}$$

$$-n_G D_G\frac{\partial C_G(x,z=z_1,t)}{\partial z}+n_G v_G C_G(x,z=z_1,t)=-n_S D_S\frac{\partial C_S(x,z=z_1,t)}{\partial z}+n_S v_S C_S(x,z=z_1,t) \tag{12}$$

Where z_1 represents the thickness of GCL, $n_i(i=G,S)$represents the porosity of the i-th layer.

2.3 Analytical solution

By applying the Laplace transform to the governing equations, the following equations can be obtained:

$$\overline{g}(s)=L(g(t))=\int_0^{+\infty} g(t)e^{-St}dt \tag{13}$$

For the GCL:

$$D_{x,G}\frac{\partial^2\overline{C}_G(x,z,s)}{\partial x^2}+D_{z,G}\frac{\partial^2\overline{C}_G(x,z,s)}{\partial z^2}-v_G\frac{\partial\overline{C}_G(x,z,s)}{\partial z}-(R_{d,G}s+\lambda_G)\overline{C}_G(x,z,s)=0 \tag{14}$$

For the SL:

$$D_{x,S}\frac{\partial^2\overline{C}_S(x,z,s)}{\partial x^2}+D_{z,S}\frac{\partial^2\overline{C}_S(x,z,s)}{\partial z^2}-v_S\frac{\partial\overline{C}_S(x,z,s)}{\partial z}-(R_{d,S}s+\lambda_S)\overline{C}_S(x,z,s)=0 \tag{15}$$

Where $\overline{C}_G(x,z,s)$ and $\overline{C}_S(x,z,s)$are the form of $C_G(x,z,t)$ and $C_S(x,z,t)$, respectively. s is the Laplace transform parameter.

Applying the Fourier series transform to equation yields the following equation:

$$\hat{F}(k)=F_c[f(z)]=\frac{2}{H}\int_0^H f(z)\cos\left(\frac{k\pi z}{H}\right)dz \tag{16}$$

For the GCL:

$$D_{z,G}\frac{\partial^2\bar{C}_G(k,z,s)}{\partial z^2} - v_G\frac{\partial\bar{C}_G(k,z,s)}{\partial z} - \left(R_{d,G}s + \lambda_G + \frac{k^2\pi^2 D_{z,G}}{H^2}\right)\hat{\bar{C}}_G(k,z,s) = 0 \qquad (17)$$

For the SL:

$$D_{z,S}\frac{\partial^2\bar{C}_S(k,z,s)}{\partial z^2} - v_S\frac{\partial\bar{C}_S(k,z,s)}{\partial z} - \left(R_{d,S}s + \lambda_S + \frac{k^2\pi^2 D_{z,S}}{H^2}\right)\hat{\bar{C}}_S(k,z,s) = 0 \qquad (18)$$

Where $\bar{C}_G(k,z,s)$ and $\bar{C}_S(k,z,s)$ are the form of $\bar{C}_G(x,z,s)$ and $\bar{C}_S(x,z,s)$ after Fourier series transform, repectivley. k is the corresponding transform parameter.

Applying the same transform to both the boundary conditions and the equations, we obtain the following equation:

For the boundary conditions:

$$C_G(k,z=0,s) = C_{in}(k,s) = f(k)g(s) \qquad (19)$$

$$C_S(k,z=z_3,s) = 0 \qquad (20)$$

For the equivalent interfacial concentration:

$$\hat{\bar{C}}_G(k,z=z_1,s) = \hat{\bar{C}}_S(k,z=z_1,s) \qquad (21)$$

For the equivalent interfacial flux:

$$n_G v_G\frac{\partial\hat{\bar{C}}_G(k,z=z_1,s)}{\partial z} = n_S v_S\frac{\partial\hat{\bar{C}}_S(k,z=z_1,s)}{\partial z} \qquad (22)$$

The homogeneous general solution of the concentration function can be written as:

$$\hat{\bar{C}}_i(k,z,s) = M_i e^{\alpha_i z} + N_i e^{\beta_i z} \qquad (23)$$

α_i, β_i an be expressed as:

$$\alpha_i,\beta_i = \frac{\left\{v \pm \sqrt{v_i^2 + 4D_{z,i}\left(R_{d,i}s + \lambda_i + \frac{k^2\pi^2 D_{z,i}}{H^2}\right)}\right\}}{2D_{z,i}} \qquad (24)$$

The matrix equation:

$$\begin{bmatrix} M_S \\ N_S \end{bmatrix} = A\begin{bmatrix} M_G \\ N_G \end{bmatrix} \qquad (25)$$

Expression for coefficient A:

$$A = \frac{1}{\alpha_S - \beta_S}\begin{bmatrix}(\gamma\alpha_G - \beta_S)e^{(\alpha_G-\alpha_S)z_1} & (\gamma\beta_G - \beta_S)e^{(\beta_G-\alpha_S)z_1} \\ (\alpha_S - \gamma\alpha_G)e^{(\alpha_G-\beta_S)z_1} & (\alpha_S - \gamma\beta_G)e^{(\beta_G-\beta_S)z_1}\end{bmatrix} \quad (26)$$

Expression for coefficient γ:

$$\gamma = \frac{n_G D_{z,G}}{n_S D_{z,S}} \quad (27)$$

Express coefficient A in matrix form:

$$A = \begin{bmatrix}A_{11} & A_{12} \\ A_{21} & A_{22}\end{bmatrix} \quad (28)$$

Translation of the concentration expression when z is zero:

$$\hat{\bar{C}}_G(k, z = 0, s) = M_G + N_G = \hat{f}(k)\bar{g}(s) \quad (29)$$

Translation of the concentration expression when z equals z_2:

$$\hat{\bar{C}}_S(k, z = z_2, s) = M_S e^{\alpha_S z} + N_S e^{\beta_S z} = 0 \quad (30)$$

The correlation between concentration expression and matrix form:

$$\begin{bmatrix}M_G \\ N_G\end{bmatrix} = \begin{bmatrix}\frac{-A_{12}e^{\alpha_S z_2} - A_{22}e^{\beta_S z_2}}{A_{11}e^{\alpha_S z_2} - A_{12}e^{\alpha_S z_2} + A_{21}e^{\beta_S z_2} - A_{22}e^{\beta_S z_2}} \\ \frac{A_{11}e^{\alpha_S z_2} + A_{21}e^{\beta_S z_2}}{A_{11}e^{\alpha_S z_2} - A_{12}e^{\alpha_S z_2} + A_{21}e^{\beta_S z_2} - A_{22}e^{\beta_S z_2}}\end{bmatrix}\hat{f}(k)\bar{g}(s) \quad (31)$$

Applying the inverse transform to the equation, the solution for the original problem is finally obtained:

For the GCL :

$$\bar{C}_G(x, z, s) = \frac{1}{2}(M_G e^{\alpha_G(k=0,s)z} + N_G e^{\beta_G(k=0,s)z}) + \sum_{k=1}^{+\infty}(M_G e^{\alpha_G z} + N_G e^{\beta_G z})\cos\left(\frac{k\pi x}{H}\right) \quad (32)$$

For the SL :

$$\bar{C}_S(x, z, s) = \frac{1}{2}(M_S e^{\alpha_S(k=0,s)z} + N_S e^{\beta_S(k=0,s)z}) + \sum_{k=1}^{+\infty}(M_S e^{\alpha_S z} + N_S e^{\beta_S z})\cos\left(\frac{k\pi x}{H}\right) \quad (33)$$

3. Model verification

To validate the effectiveness and reasonableness of the analytical solution in this study, an analytical solution for solute transport in double-layered finite porous media was chosen as a benchmark. The liner system model used in this study consists of a 1.5 mm GM, a 1 cm GCL, and a 75 cm SL. The analytical solution was validated using the one-dimensional analytical solution provided by Feng et al. (2019b). In this study, the water head was set at 0.3 m, and the other coefficients are provided in the Table 1 below.

The results, as shown in Fig.2, indicate that at the two-year mark, the pollutant concentration calculated by the model shows some differences from the data in the reference literature at distances further from the GCL. This discrepancy is attributed to the consideration of pollutant degradation within the liner in this study, resulting in lower pollutant concentrations at greater distances compared to the reference literature.

To further validate the model's accuracy in two dimensions, COMSOL Multiphysics 6.0 was used to compare the concentrations of pollutants after one year and two years. The results demonstrate a high degree of consistency between the COMSOL model and the analytical solution utilized in this study, providing robust validation for these research outcomes. The parameters used are as follows:(Ding et al., 2020; Feng et al., 2019b; Foose et al., 2002; Xie et al., 2023; Xie et al., 2014)

Table 1
Parameters used in this study

Parameter	Pollutants	GM	GCL	SL
Thickness, L (m)	–	0.0015	0.01	0.75
Porosity, n	–	–	0.7	0.3
Dry density, ρ_d (g/cm^3)	–	–	0.79	1.62
Hydraulic conductivity, k (m/s)	–	–	0.5×10^{-10}	1×10^{-7}
	–	3×10^{-13}	3×10^{-10}	8×10^{-10}
Effective diffusion coefficient, D (m^2/s)	Zn^{2+}	6×10^{-15}	7.15×10^{-10}	8.9×10^{-10}
	TOL	3×10^{-13}	3×10^{-10}	8×10^{-10}
Distribution coefficient, K_d (mL/g)	–	0	0	0
Partition coefficient, Kg	–	100	–	–

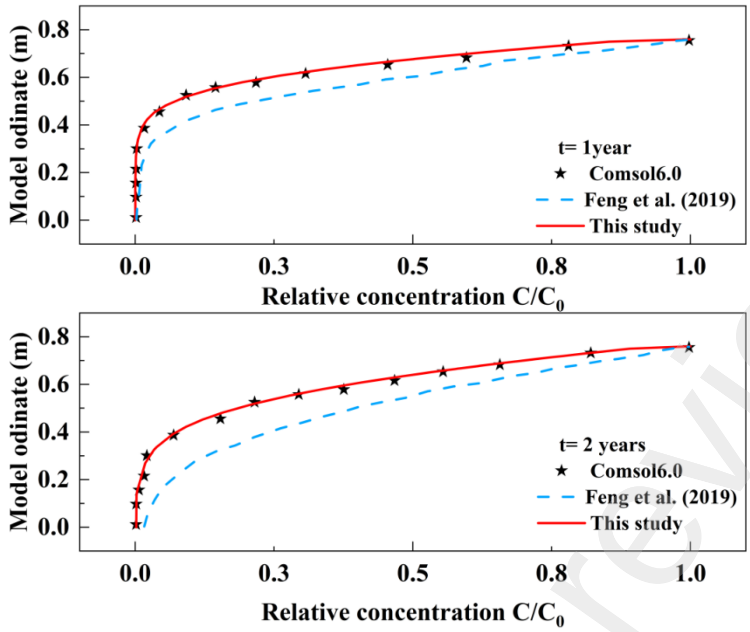

Fig. 2. Comparison of the solution in this study with the existing solution.

4. Uneven distribution of pollutant concentrations at the liner leak points

Damage to the GM in the liner system results in a non-uniform distribution of pollutant concentrations during the subsequent transport through the liner. As shown in Fig.3, the diffusion coefficient of heavy metal pollutants in the GM is significantly smaller than that in the defective areas. Therefore, this study employs distinct concentration functions for heavy metal pollutants and organic pollutants. Specifically, for heavy metal ion pollutants, this study uses the concentration function related to the width and length of the leak as proposed by Sun et al. (2022).

$$C|_{z=0} = \begin{cases} 1, 0 \le x \le \lambda \\ \zeta \frac{\partial C}{\partial z}|_{z=0} + 1, \lambda \le x \le 1 \end{cases} \tag{34}$$

Here, $\lambda = L_1/L_2$, coefficient $\zeta = \frac{D_{S,Z} L_G}{S_g D_G L_S}$, $D_{S,Z}$ represents the vertical diffusion coefficient in the soil liner, L_G represents thickness of GM, S_g represents partition coefficient, D_G is diffusion coefficient of the GM, L_S is the thickness of SL. Since heavy metal ions cannot degrade in the liner, the control equation can be simplified accordingly:

$$R_{d,i}\frac{\partial C_i}{\partial t} = D_{x,i}\frac{\partial^2 C_i}{\partial x^2} + D_{z,i}\frac{\partial^2 C_i}{\partial z^2} - v_i\frac{\partial C_i}{\partial z} \tag{35}$$

For organic pollutants, the concentration distribution can be more accurately described using the standard Gaussian function, as mentioned by Ding et al. (2020), to provide a more precise description of the concentration distribution.

$$C = C_{in,max} \times \exp(-(x-\mu)/2\sigma^2) \quad (36)$$

Where $C_{in,\ max}$ represents the largest concentration of the pollutant source, μ represents the abscissa of $C_{in,\ max}$, σ represents the distribution range of the high concentration.

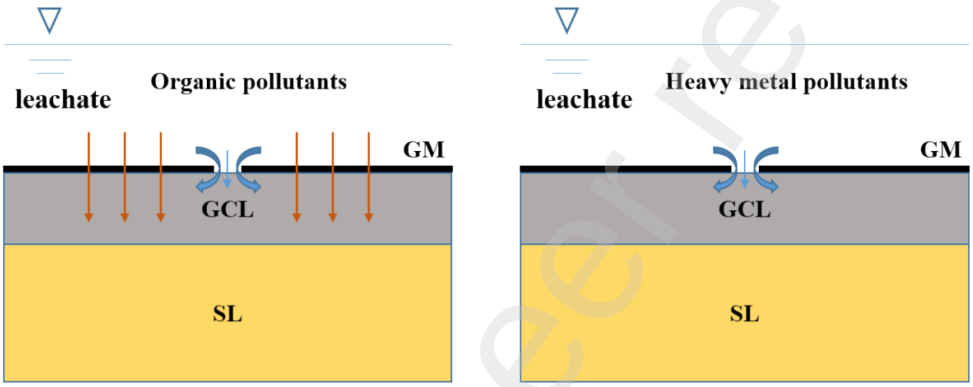

Fig. 3. Transportation process of organic pollutants and heavy metal pollutant

5. Pollution prevention performance of composite liner systems

5.1 Heavy metal ion zinc (Zn^{2+})

Zn^{2+} is common heavy metal pollutant found in leachate. Therefore, this heavy metal ion was selected for analysis. The only significant pathway for contaminant transport is through defects in the geomembrane (Foose et al., 2002). Using Eq.(34) as the initial concentration distribution function for Zn^{2+}. Fig.4 presents the breakthrough concentration of Zn^{2+} within the liner system over different time intervals. As time elapses, the breakthrough concentration of Zn^{2+} in the liner increases. However, the results of this paper are consistently slightly less than the results of Xie et al. (2023). This is caused by the differences in concentration distribution functions. As time increases, the deviation in breakthrough concentration gradually decreases. This indicates that this function can be used to describe the transport of heavy metal ions.

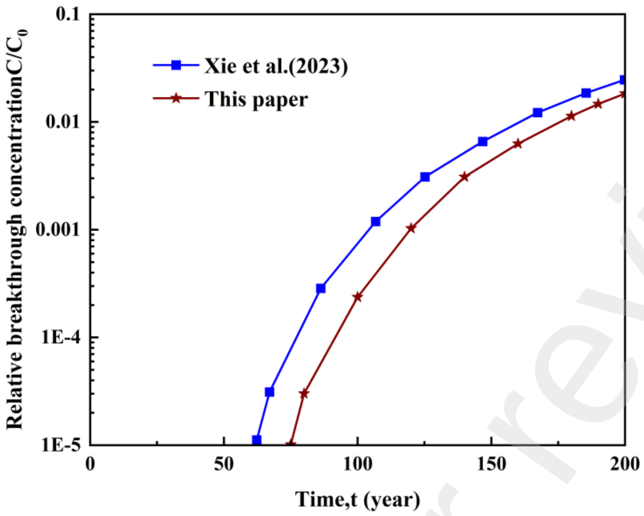

Fig. 4. Comparison of breakthrough concentration of Zn^{2+} under different time factors

5.2 Organic pollutant TOL

Leachate typically contains a substantial quantity of organic pollutants. If these organic pollutants were to leak through the GM and migrate through the composite liner system, they could cause significant damage to the soil and groundwater. Using Eq.(36) as the initial concentration distribution function, therefore, this study focuses on TOL as a representative organic pollutant to investigate its migration within the composite liner system, as illustrated in Fig.5.

Organic pollutants, such as TOL, exhibit a higher diffusion capacity within the liner compared to heavy metal pollutants, making them more likely to penetrate the GM. Due to its faster diffusion rate within the liner system, the breakthrough time of TOL less than the time of Zn^{2+}. When the migration time is short, there is a subtle difference between this study and Feng et al. (2019b). However, after 20 years, the breakthrough concentration of the two become basically consistent. These findings underscore that that function can be used to describe the transport of organic pollutants.

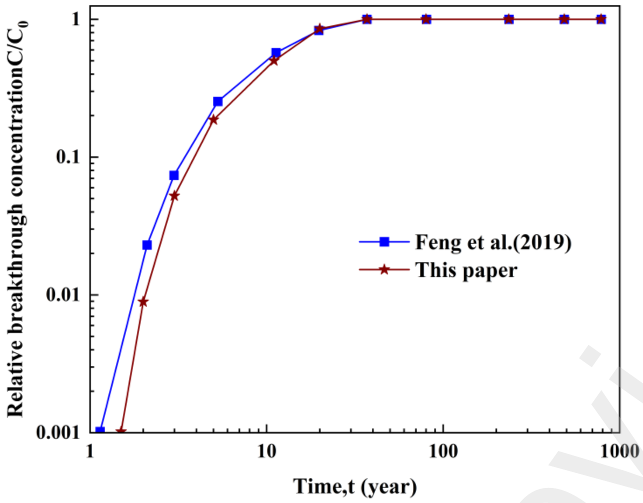

Fig. 5. Comparison of breakthrough concentration of TOL under different time factors

6. Model parameter analysis

For the GM/GCL/SL composite liner system, this study analyzed the effects of changes in SL thickness, diffusion coefficients of GCL and SL, convection coefficients, and adsorption hindrance factors on the migration of pollutants within the liner layer. The parameters of the reference model are provided in Table 1. When one parameter is changed, the other parameters are kept constant.

6.1 SL thickness

The thickness of the SL plays a crucial role in both the migration time of contaminants within the liner and the economic cost of the liner system. Understanding the appropriate thickness of the SL is therefore essential for the precise design of liner systems. To investigate this, SL thicknesses of 0.75 m, 1.5 m, 3 m, and 5 m were selected for further research and analysis.

As illustrated in Fig.6 , increasing the SL thickness from 0.75 m to 1.5 m does not significantly impact the concentration of contaminants near the GM. Instead, the concentration curve shifts upward, indicating an increase in the thickness at which the concentration becomes zero. However, as the SL thickness continues to increase beyond 1.5 m, the concentration of contaminants near the

GM remains relatively constant. For SL thicknesses of 0.75 m, 1.5 m, 3 m, and 5 m, the concentrations are essentially identical, suggesting that the SL thickness does not significantly affect the contaminant migration within the liner system.

These findings align with the work of Pandey and Babu (2017), who reported that contaminant diffusion rates stabilize beyond a certain liner thickness due to the diminishing permeability and adsorption capacity of the materials used. In contrast, Brown and Thomas (1998) found that for highly volatile organic compounds, even slight increases in liner thickness could significantly reduce diffusion rates, although their study focused on specialized industrial waste applications. Additionally, economic analyses by Sarkar et al. (2016) suggest that the cost-benefit ratio becomes unfavorable as SL thickness exceeds the optimal range, with increased material and construction costs not justifying the marginal gains in containment efficacy. This economic perspective is crucial for environmental engineering, where cost efficiency must be balanced with environmental protection.

In practice, our results suggest that a standard SL thickness of 1.5 m is sufficient for typical municipal waste containment. This recommendation supports sustainable design practices by optimizing material use without compromising liner integrity or contaminant containment capabilities.

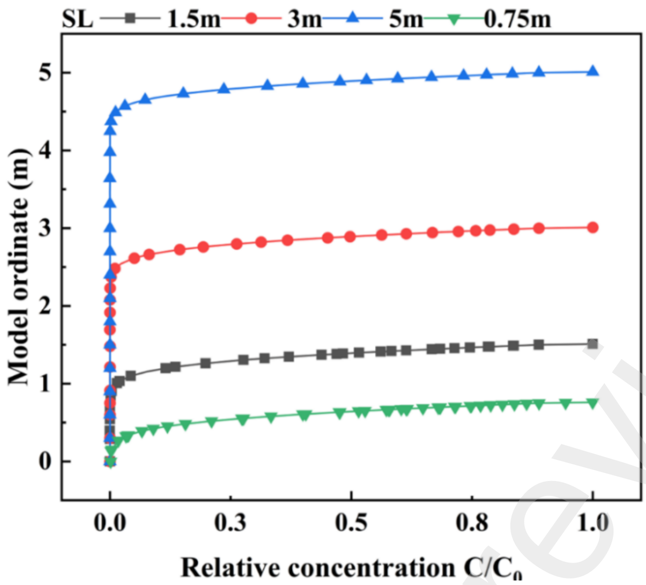

Fig. 6. The variation of pollutant concentration with the thickness of SL at different depths of coordinates.

6.2 Diffusion coefficient

The diffusion coefficient is a pivotal factor in understanding contaminant migration within a liner, reflecting the varied material properties of GCL and SL. This study investigated the impacts of different diffusion coefficients for GCL and SL on contaminant dispersion. Specifically, diffusion coefficients for GCL were considered at 3×10^{-10} m^2/s, 8×10^{-10} m^2/s, and 3×10^{-9} m^2/s; for SL, the coefficients were 8×10^{-10} m^2/s, 3×10^{-9} m^2/s, and 8×10^{-9} m^2/s.

Fig.7(a) and (b) analyze the effects of these varying diffusion coefficients on contaminant migration within the composite liner. Additionally, Fig.8 employs the COMSOL model to simulate pollutant concentrations, with sub-figures 8(a) to 8(c) highlighting the impacts of varying GCL diffusion coefficients, and sub-figures 8(d) to 8(f) showcasing those for SL. Variations in the GCL diffusion coefficient from 3×10^{-10} m^2/s to 3×10^{-9} m^2/s demonstrate a measurable influence on contaminant migration. The concentration profiles indicate that as the diffusion coefficient increases, the relative concentration of contaminants near the GM also increases. However, due to the relatively thin nature of GCL layers, this impact remains moderate. The contour plots reveal steeper concentration gradients with higher diffusion coefficients, indicating more rapid contaminant migration through the GCL. Conversely, changes in the SL diffusion coefficient result in a more pronounced increase in contaminant concentrations near the GCL interface. As the diffusion coefficient of SL increases from 8×10^{-10} m^2/s to 8×10^{-9} m^2/s, there is a significant increase in the spread of contaminants. This is attributed to the greater thickness and permeability of the SL compared to the GCL. The broader spread of contaminants with higher SL diffusion coefficients underscores the stronger influence of SL on contaminant migration within the liner system.

Xie et al. (2013) found that increases in the diffusion coefficient in similar composite liners lead to significantly enhanced migration rates of hydrophobic organic contaminants, particularly when the liners exhibit higher permeability. Moreover, studies by Anisimov et al. (2020) further corroborate that the material characteristics of SL can amplify the diffusion effects due to its greater thickness and the interaction of multiple soil layers. Interestingly, the discrepancies between the diffusion effects in GCL and SL highlighted in this study are also reflected in the work of Majumder et al. (2023), who observed that diffusion in geosynthetic layers tends to stabilize more rapidly than in soil layers, primarily due to the structured nature of geosynthetics compared to the heterogeneous composition of soil. The results suggest that careful consideration of diffusion properties is essential for designing effective composite liner systems. Optimizing the diffusion coefficients for both GCL and SL can significantly enhance the containment performance of these systems.

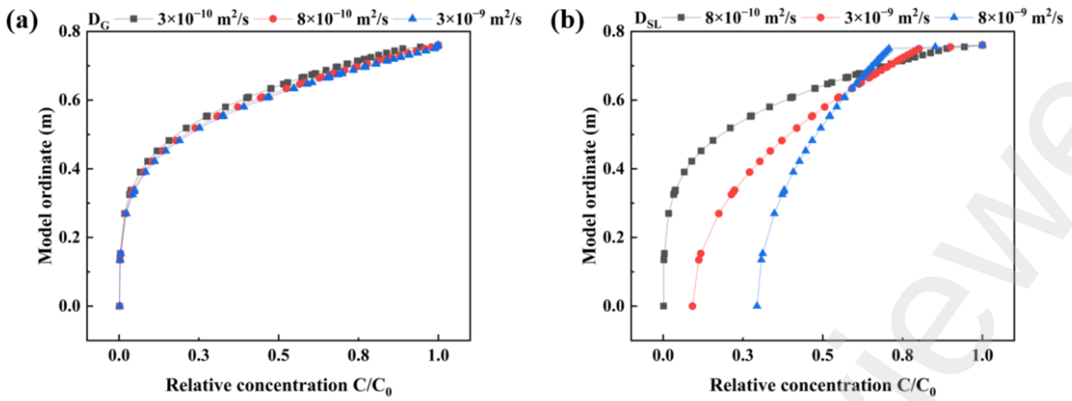

Fig. 7. The variation of pollutant concentration with the diffusion coefficient at different depths of coordinates.

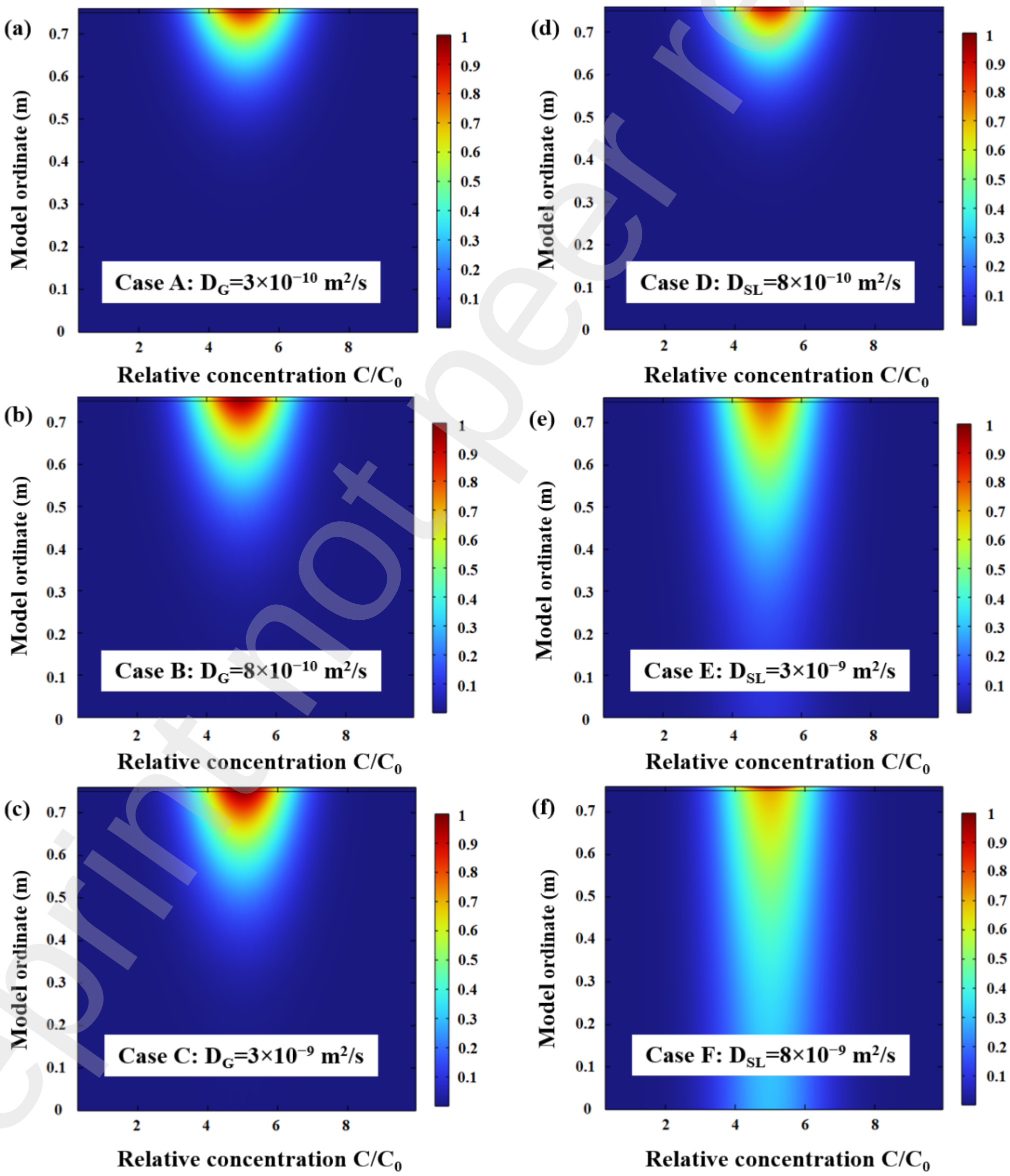

Fig. 8. Spatial distribution of pollutant concentration under diffusion coefficient:(a-c) diffusion coefficient of GCL;(d-f) diffusion coefficient of SL.

6.3 Convection coefficient

The convection process plays a pivotal role in contaminant transport within liner systems, significantly impacting both GCL and SL layers. To elucidate the role of convection in contaminant migration, convection coefficients of 1×10^{-9} m/s, 6×10^{-9} m/s, and 1×10^{-8} m/s were selected for analysis.

Fig.9 demonstrates that as the convection coefficient increases, the diffusion concentration of contaminants gradually diminishes as contaminants penetrate deeper into the liner. Specifically, Fig.10(a), 10(b) and 10(c) illustrate the effects of these varying convection coefficients as analyzed using the COMSOL model. When the convection coefficient reaches 1×10^{-8} m/s, the ccontaminant oncentration diminishes to approximately zero after migrating 0.3 m. Conversely, with a convection coefficient of 1×10^{-9} m/s, the concentration decreases to zero after migrating 0.55 m. Clearly, the convection coefficient significantly influences contaminant migration within the liner. Thus, in the practical design of landfill projects, careful consideration of the convection coefficient is imperative to ensure the rational adjustment of liner materials and design. Yeo et al. (2007) demonstrated that higher convection coefficients significantly accelerate contaminant migration in synthetic liners due to enhanced advection processes. Similarly, research by Ameijeiras-Mariño et al. (2017) in soil liners found that increases in convection coefficients could reduce the residence time of contaminants within the liner, potentially compromising the containment effectiveness unless compensated by other design modifications. In practical applications, especially in landfill project design, it is crucial to consider these convection coefficients to ensure the effective containment of contaminants by making appropriate adjustments to liner materials and system designs.

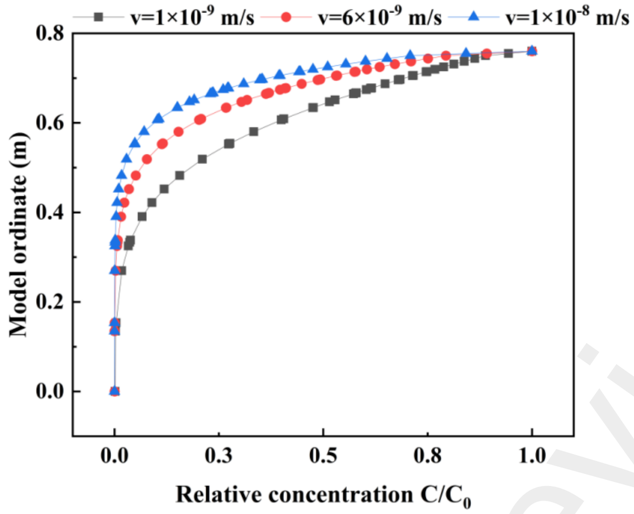

Fig. 9. The variation of pollutant concentration with the convection coefficient at different depths of coordinates.

6.4 Adsorption retardation factor

The adsorption retardation factor has a certain effect on impeding the rapid migration of contaminants within the liner layer. Adsorption retardation factors of 2, 5, and 10 were employed to simulate contaminant migration of contaminants within the composite liner. Fig.10(d),10(e) and 10(f) illustrate the effects of adsorption retardation factor as analyzed by COMSOL model.

Fig.11 demonstrates the impact of these differing adsorption retardation factor on contaminant migration. As the adsorption retardation factor increases, the migration of contaminants decelerates. For instance, with a retardation factor of 2, the contaminant concentration decreases to zero after migrating 0.25 m within the liner layer. When the retardation factor is increased to 5, the concentration drops to zero at 0.35 m. Furthermore, with a retardation factor of 10, the concentration reaches zero after migrating 0.55 m. This indicates that as the adsorption retardation factor increases, the migration of contaminants slow down, although the retarding effect on the contaminants decreases accordingly.

These findings align with the observations of Chrysikopoulos et al. (1990), who reported that the sorption effect significantly slows down pollutant migration. Additionally, studies by Lin and Yeh (2020) corroborate that the larger the adsorption factor, the shorter the migration distance of pollutants. Therefore, if the adsorption retardation effect in the liner is significant, it is essential to incorporate the retardation factor into the mathematical model to accurately predict contaminant behavior.

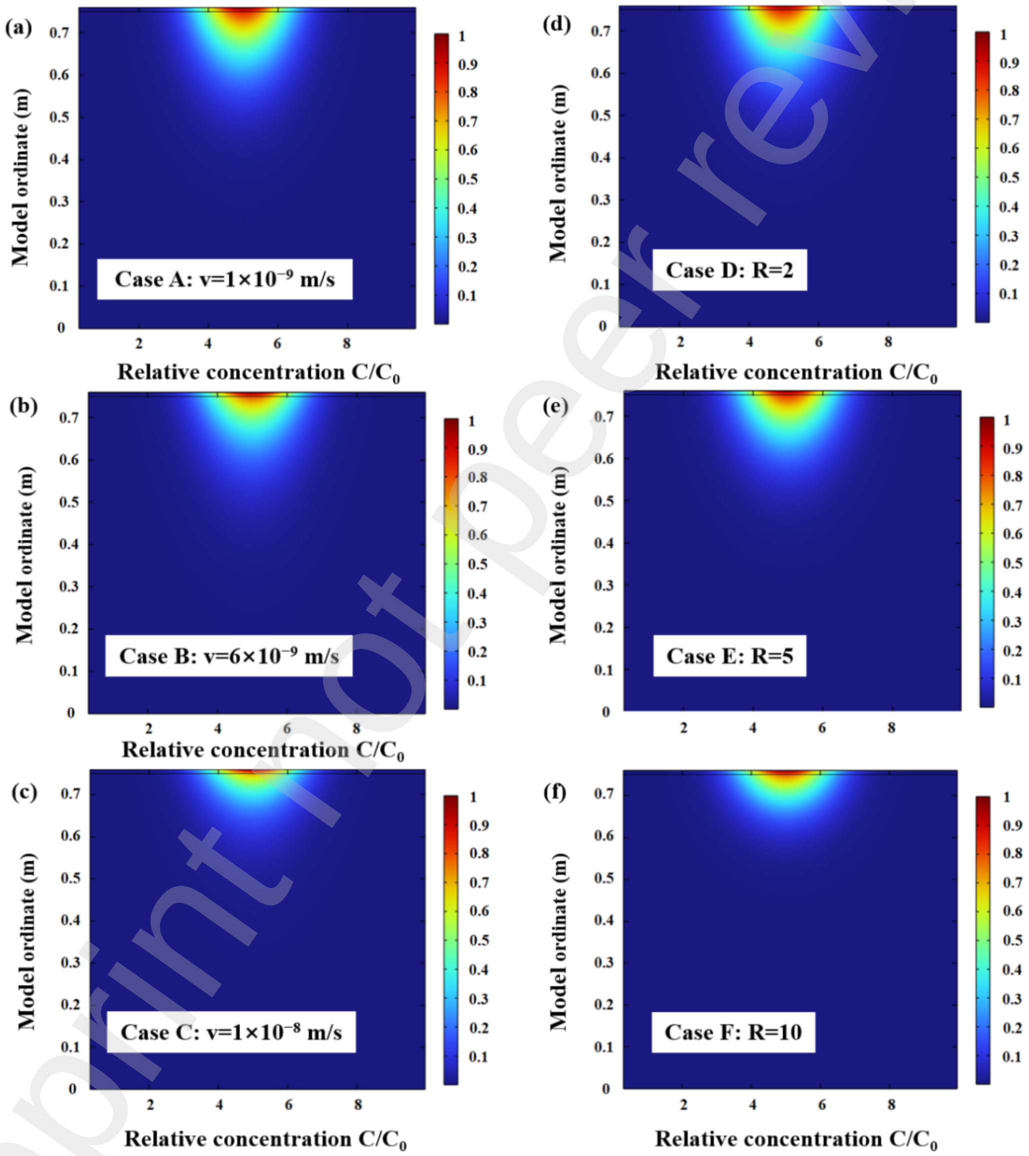

Fig. 10. Spatial distribution of pollutant concentration under convection coefficient and adsorption retardation factor: (a-c) convection coefficient; (d-f) retardation factor.

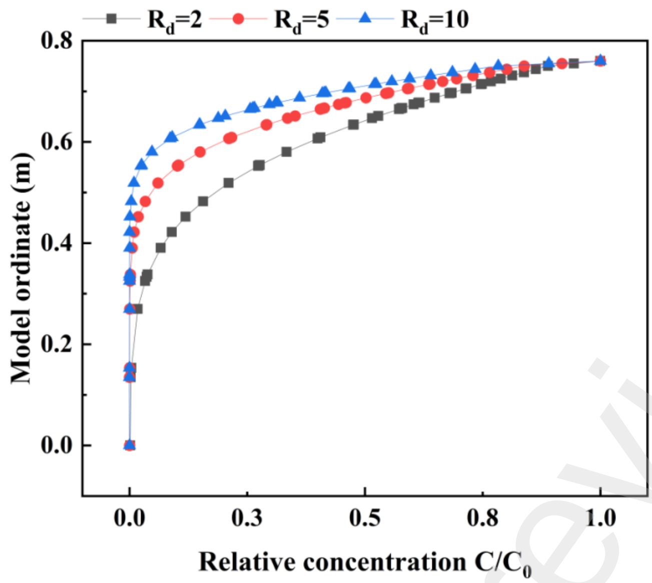

Fig. 11. The variation of pollutant concentration with the adsorption retardation factor at different depths of coordinates.

6.5 Degradation coefficient

Fig.12 illustrates the effect of the degradation coefficient of organic pollutants, considering different half-lives set at 10 years, 50 years, and 100 years. The concentrations are compared for migration times of 10 years, 50 years, and 100 years.

When the migration time (t) is 10 years, the three concentration curves exhibit minimal differences. However, as the half-life decreases, the pollutant concentration also decreases. At t = 50 years, significant differences between the concentration curves emerge, with the concentration under the 10-year half-life scenario notably lower than that under the 50-year and 100-year scenarios. The concentration is highest under the 100-year half-life scenario. As t increases to 100 years, these concentration differences become even more pronounced.

The results indicate that the half-life of organic pollutants in the composite liner system significantly affects the concentration of pollutants within the liner. However, due to the long degradation time and minimal degradation of organic pollutants over a short period, variations in degradation coefficients has a limited effect on preventing the migration of pollutants in the composite liner. Feng et al. (2019a) and Peng et al. (2021), also proposed that when the half-life is short, the degradation effect is more pronounced. However, when the half-life is long, the degradation effect can be neglected in the short term. Understanding the degradation coefficients and their impact on pollutant migration is crucial for designing effective composite liner systems. While short-term degradation may not significantly influence pollutant concentration, long-term degradation can substantially reduce contaminant levels, enhancing the liner's protective performance.

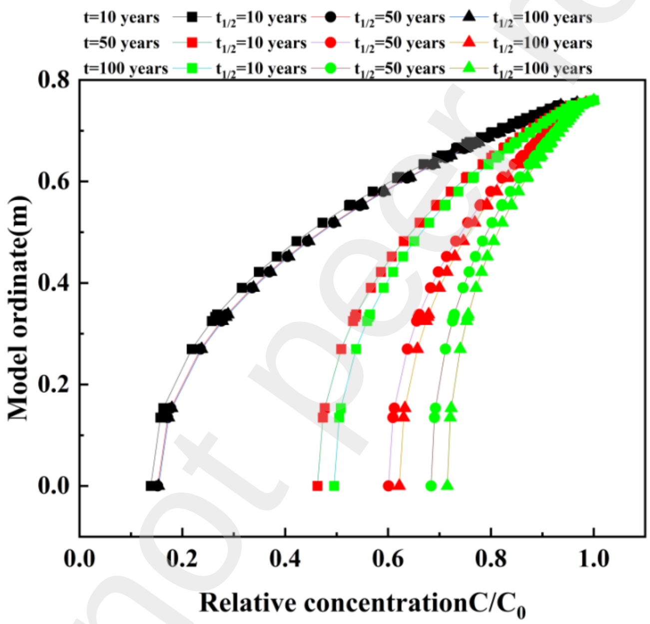

Fig. 12. The variation of pollutant concentration with the adsorption retardation factor at different depths of coordinates.

7. Limitations

One fundamental limitation of the proposed model in this study is its assumption of uniformity and isotropy within the same liner layer. This simplification overlooks the potential for heterogeneity and anisotropy, which are common in real-world scenarios. Additionally, the model does not account for the temporal changes in liner properties that can occur due to aging, chemical interactions with the leachate, or physical disturbances. Over time, the GM and other liner materials can degrade or change their properties, which can alter the effectiveness of the containment system. This temporal aspect is crucial for long-term assessments of landfill performance but is beyond the scope of the current modeling approach.

Another limitation is the exclusion of macroscopic features such as cracks or joints within the liner system, which can serve as preferential paths for the migration of contaminants. While the model assumes a defective GM, it does not specifically simulate the complex flow dynamics that can occur around these defects, nor does it consider the potential for repair or mitigation measures that might be applied in practical settings.

8. Summary

Considering the uneven distribution of pollutants behind the GM in composite liners, a two-dimensional model was developed to investigate the contaminant migration behavior. This model accounts for convection, diffusion, adsorption, and degradation processes within the liner, and has been validated through the one-dimensional analytical solution and the two-dimensional numerical results computed using the COMSOL model. Analysis of key factors led to the following conclusions:

(1) The concentration distributions of organic pollutants and metal pollutants in the liner differ to some extent, and using the same function to describe these distributions can affect the extent of contamination. Employing two distinct concentration distribution functions enhances accuracy.

(2) Compared to alternative analytical solutions and COMSOL validation results, the proposed analytical solution demonstrates a satisfactory level of accuracy, effectively describing pollutant migration processes in composite liners.

(3) The concentration curve of pollutants is more sensitive to changes in the diffusion coefficient of

SL than to changes in the diffusion coefficient of GCL. Specifically, as the diffusion coefficient of SL increases from 8×10^{-10} m^2/s to 8×10^{-9} m^2/s, the concentration curves intersect. However, when the diffusion coefficient of GCL increases from 3×10^{-10} m^2/s to 3×10^{-9} m^2/s, the concentration distribution curve of pollutants exhibits minimal changes, indicating comparable pollution prevention capabilities in both scenarios.

(4) Comparison results with the one-dimensional defective membrane GM/GCL/SL triple-layer composite liners show that in the two-dimensional case, the accumulation rate of pollutants in the liner slows down, the lateral pollutant range increases, and under the same conditions, it is more difficult for pollutants to penetrate the composite liner layer.

CRediT authorship contribution statement

Shan Zhao: Conceptualization, Funding acquisition, Supervision, Writing – original draft, Writing – review & editing. **Botao Sun**: Investigation, Methodology, Software, Writing –original draft. **Xinjia Su**: Investigation, Formal analysis, Methodology.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

Acknowledgments: The author thanks the editor and anonymous reviewers for their valuable comments on this manuscript. This study was financially supported by the National Natural Science Foundation of China (No. 42477203, No. 42177129, No.41702241) and by China Postdoctoral Science Foundation (No. 2022M720110).

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