



# Modeling Time-Dependent Behavior of Medium-Density Polyethylene Pipes

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**Abstract:** Medium-density polyethylene (MDPE) pipes are widely used for gas transmission and distribution systems. The behavior of the pipes buried in the ground is governed by soil–pipe interaction. MDPE possesses a time-dependent material property that influences the pipe–soil interaction over time. The current industry practice to account for the time-dependent effect uses secant moduli to calculate the short-term and long-term responses. This method ignores the effect of loading rate, which can have a significant effect on the pipe responses. In the current study, a rigorous finite-element (FE) analysis is employed to investigate the effect of the rate of loading on the pipe behavior. An FE modeling framework for MDPE, developed earlier by the authors, is employed in this study. Two problems, one on a conventional buried pipe subjected to ground and surface load and the other on a pipeline subjected to rate-dependent lateral ground movement, are considered. Based on the investigation, a feasible method of accounting for the time-dependent behavior of MDPE for the pipe–soil interaction problems is developed. DOI: [10.1061/\(ASCE\)PS.1949-1204.0000589](https://doi.org/10.1061/(ASCE)PS.1949-1204.0000589). © 2021 American Society of Civil Engineers.

## Introduction

Pipelines are the most efficient and common means of transporting natural gas, water, sewage, and other products from one location to another. Cast iron, ductile iron, steel, and polymers are the typical types of pipe materials used for liquid and gas transportation and distribution systems. The polyethylene/polymer pipe has become popular over the last few decades due to its various advantages, including low cost, light weight, ease of installation, and corrosion resistance. Water supply, cold water distribution, sewer, gas distribution, and irrigation are the major areas of application of polymer pipes such as high-density polyethylene (HDPE) and medium-density polyethylene (MDPE) pipes. The HDPE consists of long molecular chains without major branching, while the MDPE possesses relatively more branching (PPI 1993). Due to the chemical structure and composition, the HDPE possesses higher tensile strength, and the MDPE possesses higher shock resistance characteristics. The use of MDPE pipe is rapidly increasing in recent years for various applications.

A major challenge in predicting the behavior of the polymer pipe is its time-dependent material behavior. To account for the time-dependent behavior, the short-term and long-term values of the modulus of elasticity of pipe material is commonly employed for calculating the short-term and long-term responses, respectively (AASHTO 2010). Different approaches were proposed to express the modulus of elasticity for HDPE as a power-law function with time. Chua (1986) expressed the time-dependent relaxation modulus for HDPE pipe material as

$$E(t) = 52.6 + 460t^{0.97786} \quad (1)$$

Hashash (1991) conducted tests on corrugated HDPE pipe material and proposed the following time-dependent modulus [Eq. (2)]:

$$E(t) = 329t^{0.0859} \quad (2)$$

However, the time-dependent modulus of elasticity does not account for the strain rate–dependent behavior of the pipe material. Moore (1994) developed a linear viscoelastic model using nine kelvin elements in series for describing the viscous effect of an HDPE pipe material. This model was found to successfully simulate the stress–strain behavior of HDPE at lower strain levels (less than 1%) (Moore 1994). A nonlinear viscoelastic and viscoplastic modeling approach was then employed to reasonably simulate the stress–strain behavior under various loading conditions (Zhang and Moore 1997; Chehab and Moore 2006). Siddique and Dhar (2015) developed a strain rate–dependent nonlinear three-component elastic viscoplastic model for an HDPE pipe material. However, very limited information is currently available in the literature on the time-dependent behavior of MDPE pipe material. Das et al. (2019) and Das and Dhar (2021) conducted a comprehensive laboratory study to characterize the time-dependent behavior of MDPE pipe material commonly used in the gas distribution system. Based on the test results, Das and Dhar (2021) developed constitutive models adaptable to the framework of a widely used finite-element (FE) model, Abaqus (Dassault Systemes 2015). The modeling framework for the MDPE pipe materials is employed in the current study to investigate the time-dependent behavior of buried MDPE pipes using FE analysis using Abaqus. Two examples of buried pipe problems are considered for the investigation. The first problem is a conventional buried pipe subjected to ground and surface load. The time-dependent deflection of a buried pipe is examined. The second problem is a pipeline subjected to rate-dependent lateral ground movement. Pipelines are sometimes subjected to lateral ground movements at different rates due to landslide or fault movements due to earthquakes. The stresses developing in the pipe due to the rate-dependent ground movements are examined. Finally, a practical method of accounting for the time-dependent behavior of MDPE for the pipe–soil interaction problem is developed.

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**Table 1.** Parameters for the hyperbolic model

Hyperbolic parameters	Values
$a$	2,000
$b$	0.137
$c$	27.5
$d$	1.29

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## Time-Dependent Model

Das and Dhar (2021) conducted a detailed laboratory investigation to characterize the time-dependent behavior of MDPE pipe material. It was revealed that the stress–strain response of MDPE is nonlinear and strain-rate dependent. To account for the nonlinear strain rate–dependent behavior, a hyperbolic model proposed in Suleiman and Coree (2004) was employed [Eq. (3)]:

$$\sigma = E_{\text{ini}} \left( \frac{\varepsilon}{1 + \eta\varepsilon} \right) \quad (3)$$

where  $E_{\text{ini}}$  = initial modulus; and  $\eta$  = hyperbolic constant. The parameters are strain-rate dependent and can be obtained using the following equations (Suleiman and Coree 2004):

$$E_{\text{ini}} = a(\dot{\varepsilon})^b \quad (4)$$

$$\eta = \frac{a(\dot{\varepsilon})^b}{c + d \ln(\dot{\varepsilon})} \quad (5)$$

where  $\dot{\varepsilon}$  = strain rate; and  $a$ ,  $b$ ,  $c$ , and  $d$  = constants that can be determined by fitting with the stress–strain responses obtained from uniaxial tension or compression tests. Parameters for the models are determined based on the strain rate–dependent stress–strain relations derived from the uniaxial tensile tests, given in Table 1.

Das and Dhar (2021) also developed a framework for simulating the MDPE pipe material's creep and relaxation behavior using the features available in a commercially available FE software, Abaqus (Dassault Systemes 2015). In Abaqus, two features are available for modeling the viscous behavior of material such as the Prony series and creep law. The Prony series is based on the linear viscoelastic theory, where the elastic and viscous components are modeled as combinations of springs and dashpots. Because the nonlinear behavior was observed for the MDPE pipe material during laboratory tests, the use of the creep law was proposed. The equation of the time-hardening form of the creep law is given in Eq. (6)

$$\dot{\varepsilon}_c = A\tilde{\sigma}^n t^m \quad (6)$$

where  $\dot{\varepsilon}_c$  = creep strain rate;  $\tilde{\sigma}$  = deviatoric stress;  $t$  = total time; and  $A$ ,  $n$ , and  $m$  = power-law constants determined from curve fitting with the creep and relaxation test data (Das and Dhar 2021). The strain rate–dependent nonlinear stress–strain model and the creep law are employed in the current study for rigorous time-dependent modeling of the behavior of buried MDPE pipes. Because viscoelasticity and viscoplasticity in polymer generally occur during the deviatoric deformations (Pulungan et al. 2018; Siddique and Dhar 2015), the von Mises equivalent strains and strain rates are employed for the rate-dependent modulus of elasticity and the creep law.

## Deflections of Buried Pipes

In the design of flexible polyethylene pipes, the major consideration is to limit the deflection of the pipes under overburden and

live loads. Excessive deflections of the pipes may affect the integrity of the joints and cause excessive ground settlement. A semi-empirical equation, known as the Iowa equation (Spangler 1941), has been widely used to calculate the deflection for flexible steel pipes. The Iowa equation was developed considering bending deflection (ovaling of pipe cross section) only, as the contribution of circumferential shortening is insignificant for the steel pipes. McGrath (1998) demonstrated that the circumferential shortening is not negligible for flexible polymer pipes. He proposed a simplified equation for the deflection of polymer pipes accounting for the circumferential shortening and flexural bending, as shown in Eq. (7) (Dhar et al. 2002)

$$\frac{\Delta_v}{D} = \frac{VAF \cdot q_v}{\frac{EA}{R} + 0.57M_s} + \frac{D_l K_b q_v}{\frac{EI}{R^3} + 0.061M_s} \quad (7)$$

where  $\Delta_v$  = decrease in vertical pipe diameter (mm, in.);  $D$  = pipe diameter (mm, in.);  $VAF$  = vertical arching factor =  $0.76 - 0.71(S_h - 1.17)/(S_h + 2.92)$ ; hoop stiffness  $S_h = (M_s R/EA)$ ;  $q_v$  = overburden pressure at the springline (MPa);  $E$  = pipe material modulus (MPa);  $A$  = effective pipe wall area per unit length of pipe ( $\text{mm}^2/\text{mm}$ ), which is the same as the wall thickness for the plain pipes;  $R$  = radius of the centroid of the pipe section (mm);  $M_s$  = one-dimensional soil modulus (MPa), defined as  $M_s = (1 - \mu_s)E_s/[(1 + \mu_s)(1 - 2\mu_s)]$ , where  $E_s$  and  $\nu_s$  are the modulus of elasticity and Poisson's ratio, respectively, of the soil;  $D_l$  = deflection leg factor; and  $K_b$  = bedding coefficient.

The first term in Eq. (7) represents the average circumferential shortening. The second term represents the bending deflection, which depends on the hoop stiffness and flexural stiffness, respectively, of the pipe wall. Note that the deflection provided by Eq. (7) does not account for the installation deflection expected for the flexible pipes. During staged construction and compaction of backfill material, the flexible pipes experience deflections (Rogers et al. 1995; Masada and Sargand 2007; Dezfooli et al. 2015a; Zhou et al. 2017). Researchers suggested adding the installation deflection to that given by the design equation [i.e., Eq. (7)] to calculate the total deflection (Masada and Sargand 2007; Zhou et al. 2017). Because the installation deflection depends on the methods of placement and compaction of the backfill materials employed in each project, it would require project-specific evaluation. Therefore, the installation deflection is not considered in the current study.

For the polymer pipes with time-dependent material property, the modulus of elasticity of the material is strain rate– and time-dependent, making the deflections time-dependent. To account for the time-dependent deflections, Eq. (7) is used to calculate short-term and long-term (50 years) deflections using the short-term and long-term values of the moduli of elasticity (secant values) (AASHTO 2010). One of the major limitations in applying this approach for calculating long-term deflection is the unavailability of long-term data for estimating the long-term modulus or the pipe's behavior. Secondly, because a constant modulus of elasticity is used, the strain rate–dependent effects on the pipe responses cannot be calculated using this method. The suitability of using the method of secant modulus of elasticity and the strain rate–independent responses of MDPE pipes are examined in this study through a rigorous soil–pipe interaction analysis using the time-dependent constitutive model developed in Das and Dhar (2021).

## FE Model

Researchers commonly employ laboratory soil box tests or field tests to investigate the behavior of buried pipes and evaluate the soil–pipe interaction models they develop (Dhar et al. 2004;

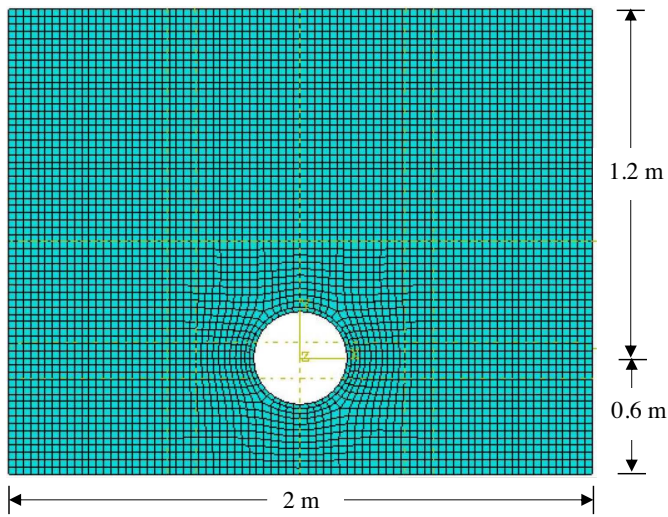


Fig. 1. FE model of buried pipe.

Dezfooli et al. 2015b; Zhou et al. 2017). The soil box test has the limitations of using rigid walls that cannot account for the effects of the native soil. On the other hand, as the native soil varies from site to site, the field test results are only applicable for a specific site condition. Dezfooli et al. (2015b) employed a numerical method introducing a trench wall around the soil box test boundary and assigning appropriate stiffness of the trench wall and boundary conditions to simulate various field conditions. Investigating the effect of bedding and the native soil is not within the scope of the current study. A pipe buried in a laboratory soil box (Dhar and Moore 2000) is investigated here to evaluate a feasible method to account for the effects of the time-dependent behavior of the pipe material on the deflections. The soil box was extensively used for studying the behavior of deeply buried pipes (Dhar and Moore 2000; Dhar et al. 2004).

A 320-mm-diameter (internal) pipe with a wall thickness of 15 mm buried at a depth of 1.2 m is investigated. A uniform pressure of 200 kPa is applied on the ground surface to simulate the earth and live load. The 200 kPa of earth pressure corresponds to a load from approximately a 10-m-high embankment. A relatively high overburden pressure is chosen to cause high deflections of the pipe for convenience in comparison. Fig. 1 shows the geometry of the soil-pipe system and the FE mesh employed for two-dimensional plane strain analysis. The thickness of the soil envelope around the pipe is larger than 2.625 times the diameter except at the bedding, where it is 1.375 times the diameter. An analysis with a larger bedding thickness (i.e., >2.625 times diameter) was also performed that showed around 5% differences in the calculated deflections.

Four-noded plane strain quadrilateral elements with hourglass control (Abaqus element type CPE4R) are used for both the pipe and the surrounding backfill soil. Two layers and one layer of elements over the wall thickness are examined and found to have an insignificant (~0.6%) effect on the calculated deflections. The nodal points along the vertical boundaries are only restrained in the horizontal direction to allow vertical movement. The nodal points along the bottom boundary have been fixed in both horizontal and vertical directions. An Abaqus/Standard module is used that employs the implicit integration scheme.

Because the focus of the current study is to examine the effect of the time-dependent behavior of pipe material, a simple linear elastic perfectly plastic (Mohr-Coulomb) model is used for the

Table 2. Creep law parameters for creep tests

Maximum stress (MPa)	A	n	m
2	$3 \times 10^{-11}$	1.825	-0.7
8.5	$3 \times 10^{-11}$	1.87	-0.7
10	$3 \times 10^{-11}$	1.89	-0.7

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soil. The soil parameters are selected as the typical values for medium to dense sand. The modulus of elasticity and Poisson's ratio of the soil are chosen as 35 MPa and 0.25, respectively, based on the measurements of stresses and strains in dense sand used in the soil box test facility (Dhar 2002). The angle of internal friction was assumed as  $42^\circ$ . However, the contribution of the shear strength parameter was insignificant for the pipe deflection. A small cohesion of 0.1 kPa was used for the sake of numerical stability.

For MDPE pipe material, the time-dependent material model developed in Das and Dhar (2021) is used, which was implemented in Abaqus through the development of a user-defined subroutine. Poisson's ratio of the MDPE is assumed to be 0.46. Nonlinear strain rate-dependent stress-strain relations [Eq. (3)] are used to calculate the pipe deflection during loading. For the time-dependent material, creep and relaxation occur at constant stress and strain, respectively. As a result, the pipe's deflection can change with time after application of the load. The pipe deflection during creep/relaxation is calculated using the creep law [Eq. (6)]. Das and Dhar (2021) used creep tests and relaxation tests to determine the parameters for the creep law model. Because the creep behavior would govern rather than the relaxation for the buried pipe, parameters obtained from the creep tests are considered. Table 2 gives the parameters obtained through fitting with creep test data (Das and Dhar 2021). It shows that A and m are the same for each stress level, while n increases with the increment of stress level. Using the magnitudes in Table 2, a value corresponding to the maximum stress level experienced by the pipe is obtained for n (i.e.,  $n = 1.82$ ) through interpolation.

As the conventional practice of pipe deflection assessment uses a linear time-dependent modulus (short-term and long-term moduli), a linear FE analysis is also performed for comparison with the results from the rigorous analysis discussed previously. The elastic moduli for linear elastic analysis are obtained at the secant value corresponding to the time, discussed later in the paper.

### Time-Dependent Responses

To calculate the responses of the pipe under the surface load of 200 kPa, the load is first applied at various rates. Then, the analyses are continued under constant load to calculate the time-dependent responses. Note that even though the applied load is constant with time, the pipe stresses can change due to the time-dependent behavior of the material. Thus, pipe behavior can be governed by the combined effect of creep and relaxation. To account for the effects, a creep law is used where the parameters for the models are selected based on creep or relaxation test data (Das and Dhar 2021), as discussed previously.

Fig. 2 shows the calculated deflections with time during the increase of load (short-term response) and during creep and relaxation (time-dependent response). The analysis was stopped at around 30,000 s to limit the computational time. Fig. 2 shows that the deflections are loading rate dependent. The vertical deflection increases [Fig. 2(a)] and the horizontal deflection decreases

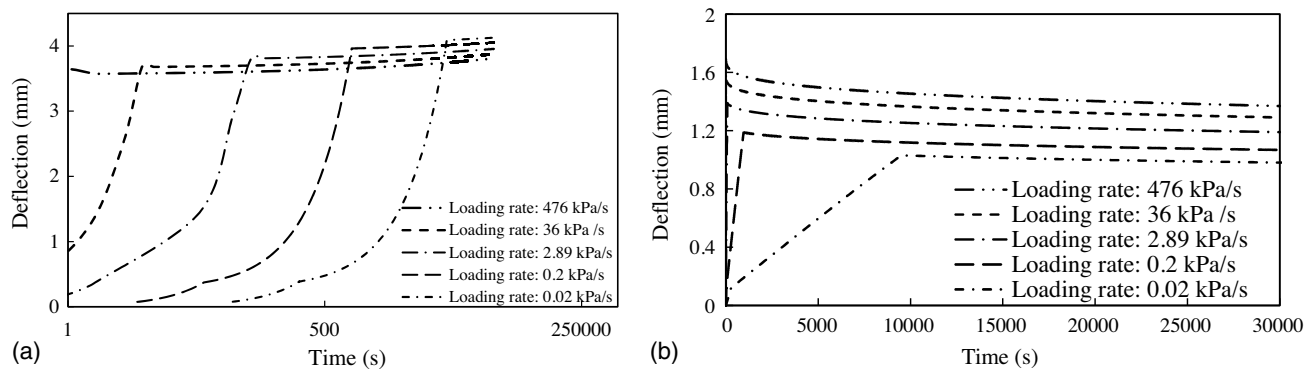


Fig. 2. Time-dependent deflections of buried MDPE pipe: (a) vertical deflection; and (b) horizontal deflection.

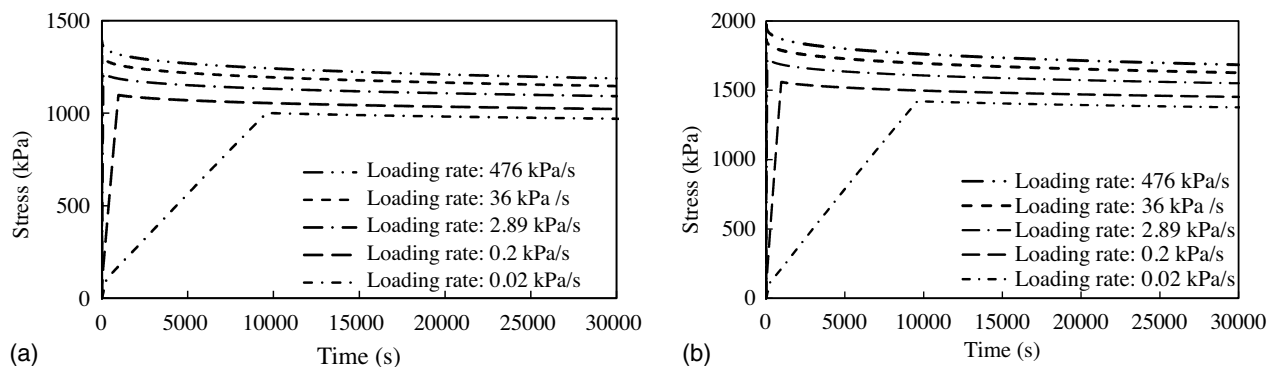


Fig. 3. Von Mises stresses in the pipes with time: (a) crown stresses; and (b) invert stresses.

[Fig. 2(b)] with the decrease of the loading rate. The opposite trends in the deflections are due to circumferential shortening, which increases the vertical deflection and decreases the horizontal direction. Beyond the loading step (short-term responses), the vertical deflections continue to increase while the horizontal deflections decrease with time (long-term responses). Note that the magnitudes of the long-term deflection are higher when the short-term deflections are higher. The constant short-term and long-term moduli of elasticity (recommended in the design codes) cannot be used to calculate these deflections. In Fig. 2(a), the time is presented in logarithmic scale to show the rate of increase of vertical deflections with time because the calculated increase of deflection is small (and not well visible in normal scale) within the time considered (i.e., 30,000 s  $\approx$  8.5 h).

The von Mises stresses in the pipe wall at the crown and the springline are investigated to examine the stress relaxation due to the time-dependent effects. Fig. 3 plots the calculated von Mises stresses with time. It reveals that even though the vertical deflections are higher and horizontal deflections are lower at lower loading rates (Fig. 2), the stresses at both the crown and the springline decrease with the decrease of loading rate. The stresses are reduced further with time under the constant applied pressure, indicating stress relaxation behavior. Thus, the long-term performance of the pipes appears favorable in terms of pipe wall stresses (and hence strain). Long-term deflection is the major concern for the performance of the pipe. The time-dependent deflection of the pipes is, therefore, further examined.

The aforementioned study presents a rigorous time-dependent soil–pipe interaction analysis to calculate the responses of buried pipe, including deflections. However, the rigorous FE analysis is

often prohibitive for engineering design. A simplified analysis using a constant time-dependent modulus of elasticity is examined to calculate time-dependent deflections.

### Short-Term Deflections

Current design code (i.e., [AASHTO 2010](#)) recommends using a short-term modulus to estimate the short-term deflection (immediately after application of the loads). However, as discussed previously (Fig. 2), the short-term deflection significantly depends on the rate of loading. The suitability of using a rate-dependent modulus in calculating the rate-dependent deflections is examined here. The rate-dependent initial modulus is estimated using Eq. (4) based on the strain rates calculated from the FE analysis. Although the strain rate–dependent stress–strain response is nonlinear, a linear stress–strain relation (i.e., a constant modulus of elasticity) can be assumed at low strain levels ( $<0.01$ ) for MDPE ([Das and Dhar 2021](#); [Bilgin et al. 2007](#)). In the current study, the maximum strain in the pipe is calculated to be less than 0.0045. Therefore, a constant strain rate–dependent modulus of elasticity (initial tangent modulus) can be used. Using the constant modulus, the deflections of the pipe are calculated using a linear FE analysis. The calculated deflections are compared with those obtained using the time-dependent analysis (discussed previously) in Table 3. It reveals that both horizontal and vertical deflections calculated using the strain rate–dependent constant modulus matches well (within around 2%) with those calculated using the rigorous time-dependent analysis. Thus, the rate-dependent constant modulus can reasonably be used to calculate the short-term deflections if the strain rate within the pipe can reasonably be estimated. For the MDPE material, the

**Table 3.** Comparison of initial deflections

Loading rate (kPa/s)	Calculated strain rate (s)	Initial modulus (MPa)	Deflection using time-dependent analysis (mm)		Deflection using constant initial modulus (mm)	
			Vertical	Horizontal	Vertical	Horizontal
476	0.01	1,064	3.57	1.69	3.62	1.70
36	0.0016	828	3.68	1.55	3.70	1.58
2.89	0.00012	580	3.80	1.38	3.84	1.40
0.2	0.000008	400	3.96	1.18	4.02	1.19
0.02	0.000001	325	4.09	1.03	4.12	1.06

following equation is proposed to calculate the strain rate–dependent short-term modulus:

$$E_0 = 2000(\dot{\epsilon})^{0.137} \quad (8)$$

where  $E_0$  = short-term modulus in MPa; and  $\dot{\epsilon}$  = strain rate per second.

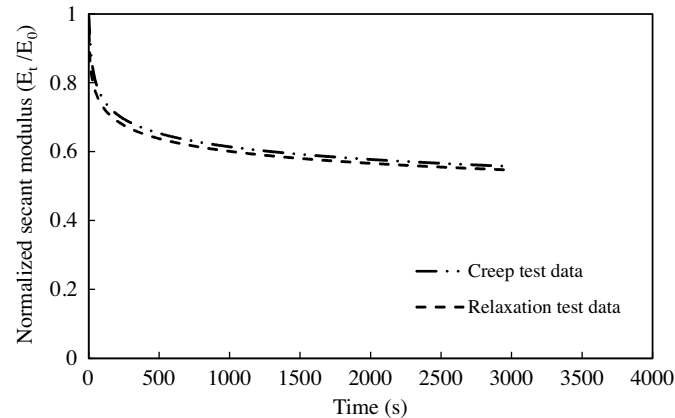
### Time-Dependent Deflections

For the time-dependent deflections, the use of a constant time-dependent modulus is examined against the results from the rigorous time-dependent analysis. The time-dependent deflections under the constant applied pressure are governed by the creep and relaxation behavior of the material. As reported in Das and Dhar (2021), the creep behavior and relaxation behavior of the MDPE material significantly depends on the stress levels and deformation levels. Thus, the stress-specific creep or strain-

specific relaxation parameters would be required to simulate the time-dependent deflection. Because the calculated stresses and strains are less (as discussed previously), the test data corresponding to the lowest stress and the lowest strain is used to estimate the modulus of elasticity of the material for calculating time-dependent deflections.

With the creep data, the applied stress is divided by the time-dependent strains, and with the stress relaxation data, the time-dependent stresses are divided by the applied strain to calculate the time-dependent elastic secant modulus ( $E_t$ ). The calculated elastic moduli are normalized by the corresponding initial value ( $E_0$ ) and plotted in Fig. 4. The figure shows that the normalized elastic moduli obtained from the creep and relaxation test data for the particular stress and strain levels are close to each other. The normalized modulus can be represented using a power-law model, as in Eq. (9)

$$\frac{E_t}{E_0} = 0.9582t^{-0.07} \quad (9)$$

**Fig. 4.** Normalized time-dependent secant moduli.

The normalized modulus [Eq. (9)] can be used to estimate the elastic modulus at any time through multiplying by the initial modulus (strain-rate dependent) of the material. Thus, the long-term modulus is dependent on the initial modulus, accounting for the effect of short-term deflections on the long-term deflections. Deflections calculated using FE analysis with the constant time-dependent modulus are compared with those from the rigorous time-dependent analysis in Table 4. The deflections calculated using the simplified FE analysis using a constant elastic modulus are within 3% of the deflections calculated using rigorous time-dependent FE analysis. Thus, the simplified approach of FE analysis (based on a constant elastic modulus) can reasonably be applied to calculate the time-dependent or long-term deflections. Considering that the stress levels expected for buried pipe are typically less (Bilgin et al. 2007), Eq. (8) can be used to calculate the time-dependent elastic modulus for estimating the long-term deflections using the power law equation [Eq. (9)].

**Table 4.** Comparison of time-dependent deflections

Time	Vertical deflection (mm)		Horizontal deflection (mm)	
	Using time-dependent elastic modulus	Rigorous time-dependent analysis	Using time-dependent elastic modulus	Rigorous time-dependent analysis
0	3.62	3.57	1.7	1.69
5,000	3.85	3.71	1.4	1.50
10,000	3.87	3.75	1.36	1.45
15,000	3.88	3.77	1.34	1.43
20,000	3.898	3.78	1.33	1.4
25,000	3.9	3.79	1.32	1.38
30,000	3.91	3.8	1.31	1.37

### The Proposed Method of Deflection Calculation

The aforementioned study reveals that the short-term and long-term pipe deflections calculated using the rigorous time-dependent models can reasonably be obtained using an equivalent linear model (with a constant modulus of elasticity). However, the short-term modulus of elasticity is strain-rate dependent. The long-term modulus of elasticity also depends on the initial stress/strain levels. The following methods are proposed for calculating the deflections of buried MDPE pipes, accounting for the strain rate–dependent short-term modulus and stress-dependent long-term modulus:

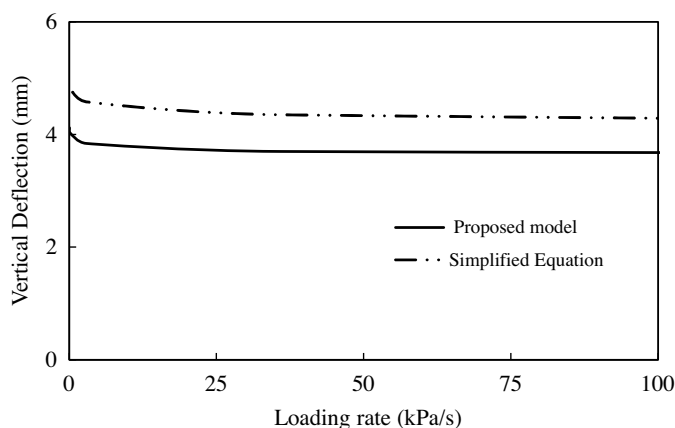
- For short-term deflection, Eq. (8) can be used to calculate the strain rate–dependent modulus of elasticity of the pipe material. To estimate the strain rates during design, the maximum strain corresponding to an applied load can be first calculated using a constant modulus. Then, the rate of strain can be calculated through dividing the maximum strain by the duration for the application of the load (i.e., construction period).
- For long-term deflection, Eq. (9) can be used to calculate the time-dependent modulus, where the initial modulus ( $E_0$ ) is the short-term modulus calculated in the previous step.

### Evaluation of the Simplified Design Equation

The deflections obtained from the FE analysis are compared with those calculated using the simplified design equation [Eq. (7)] for evaluation. The deflection lag factor  $D_i$  and the constant  $K_b$  in Eq. (7) are assumed as 1 and 0.1, respectively, as suggested in Zhou et al. (2017). All other parameters are as those used in the FE analysis. Fig. 5 compares the calculated short-term deflections with the various rates of loading. It shows that the simplified design equation overestimates the vertical deflections with respect to the FE calculations. The maximum overestimation was around  $\sim 16.4\%$  at the lowest loading rate.

### Pipelines Subjected to Lateral Ground Movement

Pipelines buried underground are often exposed to various hazards, including differential ground movement resulting from natural disasters (e.g., landslide, earthquake, etc.) and human activities (e.g., construction, mining, tunneling, etc.) (O'Rourke 2010). The ground movements have been identified as one of the significant causes of pipeline failure (CEPA 2017). The pipelines can be subjected to longitudinal, lateral, and/or oblique ground loads depending upon ground movement orientations. The ground movement



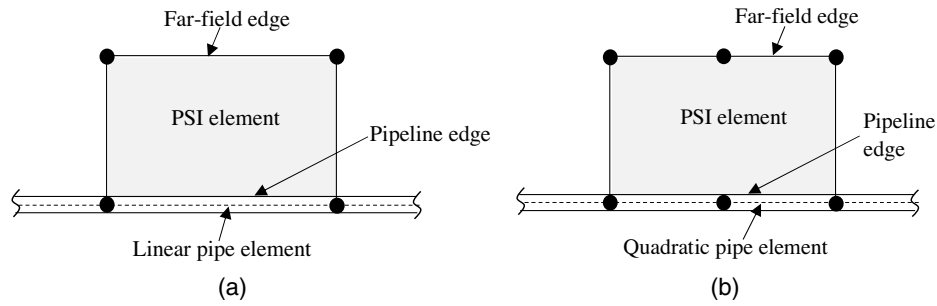
**Fig. 5.** Comparison of deflections from FE analysis and simplified design equation [Eq. (7)].

effects on buried pipelines were extensively studied with particular attention to strike-slip fault movements. Researchers employed analytical (Karamitros et al. 2007; Ni and Mangalathu 2018) and numerical (Yimsiri et al. 2004; Vazouras et al. 2015) methods to calculate wall strains for pipelines crossing the strike-slip faults. The mechanical responses of HDPE pipelines subjected to fault motions were studied in Ha et al. (2008), Xie et al. (2011), Naeini et al. (2016), and others. The time-dependent behavior of the pipe materials was not explicitly investigated in these studies. Pipelines are also subjected to transverse permanent ground deformation having distributed profiles (Cubrinovski et al. 2011; Ni et al. 2018). Studies on the pipelines subjected to distributed ground deformation are very limited. A problem of pipelines subjected to distributed lateral ground movement is investigated here.

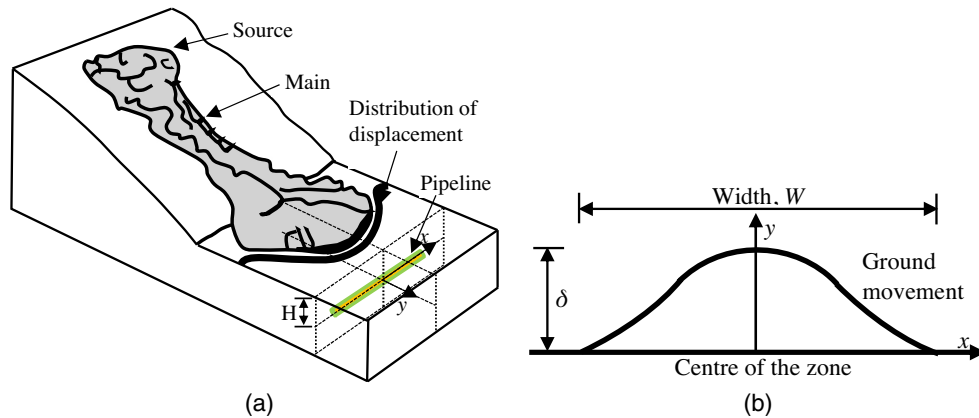
Early research on pipelines subjected to a lateral ground movement focused on identifying the loads on the pipe. Audibert and Nyman (1978) conducted tests with steel pipes of different diameters (25, 60, and 114 mm) and observed the soil load against the pipe's lateral displacements in sand. Trautmann and O'Rourke (1983) also conducted full-scale experiments and proposed lateral forces resulting from relative movement between the pipe and the surrounding soil under plane strain conditions. Based on these studies, load-displacement relations were developed, which are used as the basis of calculating spring constants to represent soil–pipe interaction for the analysis of pipelines. The current industry practice for pipe stress/strain assessment is to idealize the pipeline as a series of beams and model the soil–pipe interaction using Winkler springs. ALA (2001) recommends the spring parameters to represent soil resistance in lateral, axial, and vertical directions. Almahakeri et al. (2012, 2014) conducted large-scale tests with steel pipes and glass fiber reinforced polymer (GFRP) pipes to experimentally examine the bending behavior of the buried pipes. They employed three-dimensional FE analysis to simulate the pipe responses numerically, where the soil was idealized as an elastoplastic continuum. However, the continuum-based three-dimensional FE analysis is impractically time-consuming (Ni et al. 2018). In this regard, a simplified FE model idealizing the pipes as beam-type structures surrounded by a Winkler spring is more suitable for engineering analysis and design. Beam-spring type of analysis is, therefore, performed here for the stress/strain assessment of MDPE pipes considering the rate-dependent material properties. However, the effects of ovalization of the pipe cross section and resulting stress distribution cannot be accounted for using this method.

### Pipe–Soil Interaction Element

The pipe–soil interaction (PSI) element available in Abaqus, the FE analysis software, is used to idealize the soil as a Winkler media. The PSI element is a special type of element that interacts with the structural beam element, as shown in Fig. 6. One side of the PSI element shares nodes with the nodes of the pipe element, which is a beam-type element. The other side of the PSI element represents a far-field surface, where the boundary condition (i.e., ground movement) can be applied. The number of nodes on the side sharing the pipeline matches the number of pipe/beam element nodes. Thus, there are two nodes per side (four total nodes) for a linear pipe element and three nodes per side (six total nodes) for the quadratic pipe element (Fig. 6). Abaqus/Standard provides four-noded and six-noded two-dimensional elements (PSI24 and PSI26) and three-dimensional elements (PSI34 and PSI36) for modeling soil–pipe interaction. Each node of the element has only one displacement degree of freedom.



**Fig. 6.** Pipe–soil interaction model: (a) four-noded PSI element; and (b) six-noded PSI element.



**Fig. 7.** Pipeline perpendicular to ground movement: (a) direction of ground movement; and (b) distribution of ground movement.

The deformation in the PSI element is defined as the relative displacements between the two edges of the element. If the relative displacement is greater than zero, forces are applied to the pipeline nodes. The applied forces can be a linear (elastic) or nonlinear (elastic-plastic) function of the strains, defined by

$$\varepsilon_{ii} = \Delta u \cdot e_i \quad (10)$$

where  $\Delta u = u^f - u^p$  is the relative displacement between two edges ( $u^f$  is the far-field displacement and  $u^p$  is the pipeline displacement); and  $e_i$  = local direction vector. A suitable constitutive model is required to calculate the nodal forces from the strains. The constitutive relation of PSI elements is usually determined based on experimental results, which is expressed as a force per unit length along each of the orthogonal directions. Data for a linear or a multi-linear model can be provided as tabular input in Abaqus.

### FE Modeling

Ground movement due to landslide can impact the pipelines in many different ways (Argyrou et al. 2018). A pipeline crossing a landslide perpendicular to the general direction of soil movement (Fig. 7) is considered in the present study. Distribution of the ground movement over the length of the pipe affected by the landslide is shown in Fig. 7(b). The ground deformation is the maximum at the center of the affected zone and the minimum near the margins. The length of the affected zone can vary from several meters to over kilometers. A landslide length of 8 m is considered in the present study to demonstrate the effect of the time-dependent pipe material's behavior.

Two-dimensional FE analysis is performed for investigation of the pipe subjected to the lateral ground movement. A type of beam element (PIPE21 in Abaqus) is used to idealize the pipe, and the element PSI24 is used to model the soil–pipe interaction. The displacements corresponding to the ground movement are applied to the pipe. Researchers employed cosine functions to approximate the type of ground movement shown in Fig. 7(b) (O'Rourke 1989; Suzuki et al. 1988; Ni et al. 2018). The deflections given by the cosine function shown in Eq. (11) are applied in the perpendicular direction of the pipeline

$$y(x) = \delta \left[ \cos \frac{\pi x}{W} \right]^2 \quad (11)$$

where  $y(x)$  = ground displacement at a distance  $x$  measured from the center of the ground movement zone;  $W$  = width of the zone; and  $\delta$  = peak ground displacement (at the center). The power of the cosine term (i.e., 2) in the equation accounts for the spread of the area, with a smaller power corresponding to a greater spreading.

Fig. 8 shows the FE mesh used in the analysis. A pipe length of 10 m is modeled, which is 2 m greater than the width of the ground displacement zone. The pipe length was found to be sufficient through comparison with the results of the analysis with longer pipes. The pipe is discretized with a uniform element size of 0.01 m. Hinge support is applied at the two ends of the pipeline.

For the MDPE pipe material, the rate-dependent material model developed in Das and Dhar (2021) is used. Poisson's ratio of the MDPE is assumed to be 0.46. A bilinear (elastic-perfectly plastic) constitutive model is used for the PSI elements to model the

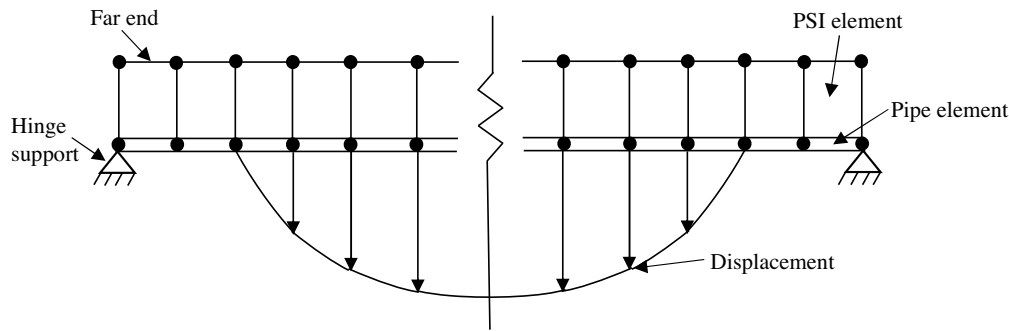


Fig. 8. FE mesh for the analysis of ground movement (schematic).

Table 5. Parameters considered for the analysis of pipe subjected to ground movement

Item	Parameter	Numerical model
Pipe	Diameter, $D$ (mm)	110
	Wall thickness, $t$ (mm)	6.3
	Material	Time-dependent model (Das and Dhar 2021)
Ground displacement	Peak ground displacement (m)	0.6
	Width of ground movement zone (m)	8.1
Springs	Axial resistance (kN/m)	12.38
	Axial elastic displacement (mm)	8
	Lateral resistance (kN/m)	31.21
	Lateral elastic displacement (mm)	8

nonlinear soil–pipe interaction. The parameters for the constitutive model are selected based on a previous study (Luo et al. 2014). Table 5 gives the detailed parameters considered in this study.

### Time-Dependent Responses

To investigate the effects of the rate of ground movement on the pipe responses, the deformation is applied at five different rates: 1.5, 0.15,  $1 \times 10^{-2}$ ,  $1 \times 10^{-3}$ , and  $1.5 \times 10^{-5}$  m/s. The maximum pipe stresses calculated due to rate-dependent ground movement are illustrated in Fig. 9. This figure shows that the pipe stress increases with the increase of ground displacement. The rate of increase of the stress is higher for higher rates of ground displacement. At the peak displacement of 0.6 m, the maximum pipe stress increased from 6.9 to 14.3 MPa (more than double) for an increase of the ground movement rate from  $1.5 \times 10^{-5}$  to 1.5 m/s. Thus, the buried MDPE pipe can experience stress as high as its allowable limit, depending on the size of the landslide and the rate of the ground movement. However, the strain on the pipe wall during the ground movement process may not be significant. Fig. 10 shows the maximum pipe wall strains against ground displacement for different movement rates. As in pipe wall stress, the maximum strains occur at the midlength of the pipe. For the range of ground movement rate considered, the maximum pipe wall strain ranges from 3.9% to 4.7% at the peak displacement of 0.6 m. The smallest strain is for the highest rate of ground movement, unlike the stress. The stress was the maximum for the highest rate of ground movement. Note that the effect of the ground movement rate on the pipe wall strain is less significant (the difference is  $\sim 17\%$ ) than the pipe wall stress.

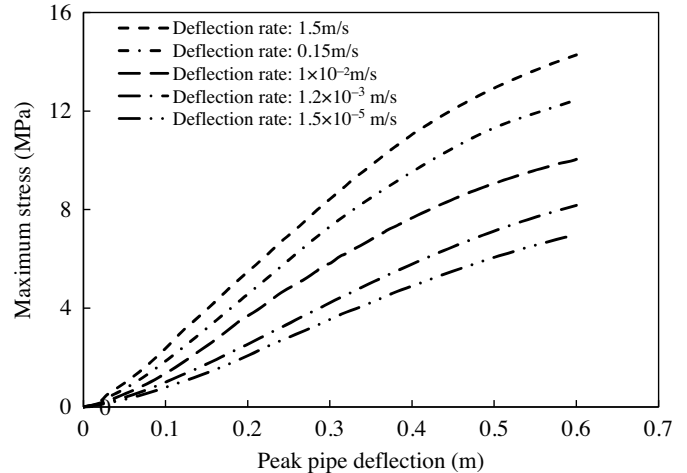


Fig. 9. Maximum stress at different rates of ground movement.

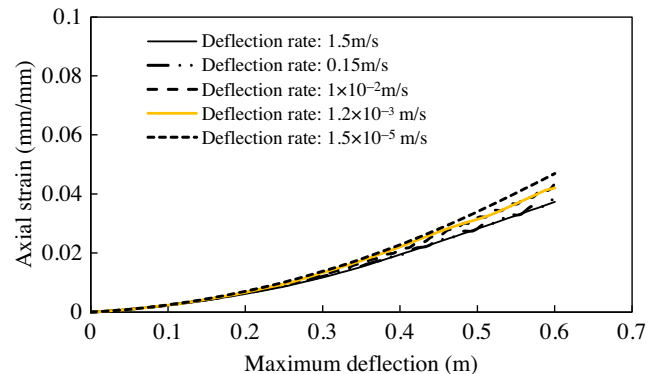
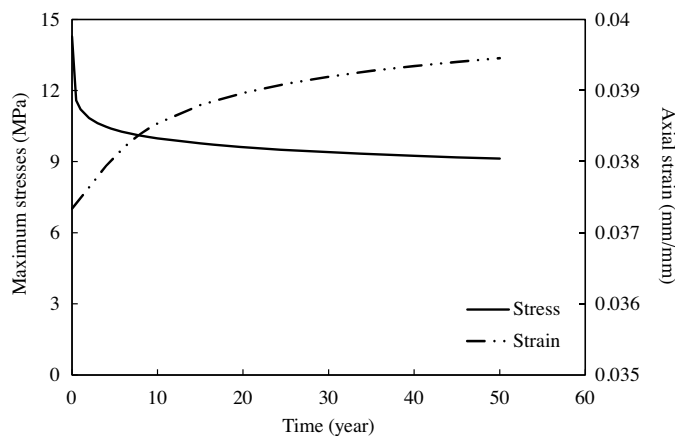


Fig. 10. Maximum strain at different rates of ground movement.

The pipe stress and strain experienced by MDPE pipe after the incident of ground movement will change with time due to the time-dependent property of the material. The changes in the maximum stress and strain corresponding to the highest ground movement rate are calculated using the proposed power-law model for MDPE pipe shown in Eq. (9). Fig. 11 shows the changes in stress and strain over a period of 50 years. The stress is found to reduce by about 35% in 50 years since the incident of ground movement (i.e., reaching the maximum displacement of 0.6 m). However, the





**Fig. 11.** Time-dependent responses of the deflected pipe.

pipe wall strain continues to increase with time. In 50 years, the strain was increased by 5.66%.

## Conclusions

In this paper, the time-dependent behavior of buried MDPE pipe is investigated considering a conventional buried pipe problem under vertical load and a pipeline subjected to rate-dependent lateral ground movement. A rigorous modeling technique and an equivalent simplified method (using a secant modulus) were employed to recommend a practical approach to account for the time-dependent effects during analysis. The major findings of this study are as follows:

1. The rigorous time-dependent modeling technique can be used to investigate the responses of buried pipes having time-dependent material properties.
2. For the pipe under vertical load, the pipe's vertical deflection is higher, and the horizontal deflection is less for a slower loading rate. Beyond the loading stage, the vertical deflection increases and the horizontal deflection decreases with time under a fixed applied pressure. However, the pipe wall stress is less for the slower loading rate that reduces further with time under the constant load. Thus, long-term vertical deflection is the primary consideration for the design of the pipe.
3. The rate-dependent constant modulus proposed by the authors can reasonably be used to calculate the short-term deflections during the application of loads. For calculation of the long-term deflection (under a fixed applied pressure), a time-dependent secant modulus can be used. However, the time-dependent soil modulus depends on the initial stress level in the pipe. An equation for time-dependent normalized modulus is proposed for calculating the secant modulus at a given time ( $t$ ). Based on these observations, simplified methods are proposed for calculating the short-term and long-term deflections of buried MDPE pipes using a constant elastic modulus.
4. The existing simplified design equation is found to overestimate the deflections of MDPE pipes.
5. For the pipes subjected to lateral ground movement, stresses experienced by the pipe are higher for a higher rate of ground movement. However, the pipe strain is less for the higher ground movement rates. The effect of the ground movement rate on the increase of the stress is also significantly higher than the effect on the decrease of pipe wall strain.

6. The pipe wall stresses reduce and the pipe wall strain increases with time since the incident of ground movement.

## Data Availability Statement

Some or all data, models, or codes generated or used during the study are available from the corresponding author by request. These include Abaqus input files and Excel data files.

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