NLP Optimization Model as a Failure Mechanism for Geosynthetic Reinforced Slopes Subjected to Pore-Water Pressure

Primož Jelušič[1]; Bojan Žlender[2]; and Bojana Dolinar[3]

Abstract: The majority of slope failures are triggered by excessive rainfall and the consequent increase in pore-water pressure within the slope. This paper presents the results of a computer code that quantifies earth pressure coefficients. This code is based on limit-equilibrium analyses and is used for the internal design of geosynthetic reinforced soil structures and to identify the critical failure mechanism. The critical failure mechanism is the largest value of out-of-balance force. For this purpose, the nonlinear programing (NLP) approach was used, and a NLP optimization model, TMAX, was developed. The model was used for failure mechanisms, assuming that the failure surfaces were bilinear. The influence of pore-water pressure on the potential failure surface was analyzed. The model was developed under basic principles. Optimally, the system is best suited for structures with varying geometries, different backfill unit weights, varying types of soil shear resistance, and different pore-water pressures. A numerical example was used to demonstrate the effect of pore-water pressure on the required strength of reinforcement and on the efficiency of the introduced optimization approach. **DOI: 10.1061/(ASCE)GM.1943-5622.0000604.**

Author keywords: Reinforced soil; Limit-equilibrium analysis; Seepage; Nonlinear programing (NLP).

Introduction

Geosynthetic reinforced slopes consist of compacted soil embankments with geosynthetic layers of tensile reinforcement to enhance stability. The reinforcement holds together the soil mass from both sides of the surface, which thus increases the safety factor of the existing slope. Most slope failures are triggered by intense precipitation and severe flooding, which increases the pore-water pressure within the slopes. Pore-water pressures are likely to vary during the design life of the structure and are relatively less well controlled than other parameters. The use of geosynthetic reinforcements installed in the embankment contact zone can enhance slope stability. If the calculated stability of the slope is inadequate, the reinforcement can cause an additional resisting force in the equilibrium equations.

Research published in this field provides analytical, numerical, and experimental information as well as many case studies on geosynthetic reinforced slopes. Generally, two different approaches have been developed to calculate slope stability, which include reinforcement forces. The first approach is called the limit-equilibrium method because the safety factor is based on statics that consider the force and/or moment equilibrium. Numerous limit-equilibrium methods are available (Hopkins et al. 1975; Duncan 1996). Wright and Duncan (1991) split the sliding mass into slices and considered the stability of each slice in turn (slices method). The limit-equilibrium method is used to calculate the horizontal force due to lateral earth pressure. This pressure is supported by reinforcement layers to ensure structure equilibrium (Schmertmann et al. 1987; Leshchinsky and Boedeker 1989; Jewell 1989; Huang 1986). The second approach for stability analysis is the finite-element method, based on solid mechanics. This method considers both equilibrium and compatibility equations (Huang 2014). Numerical methods are used to locate critical shear surfaces, where the lowest factor of safety prevails. Despite more sophisticated numerical models (Liu and Zhao 2013; Bai et al. 2014), the limit-equilibrium methodology is still widely applied (Abramson 2002; Bathurst et al. 2008; Vieira et al. 2011; Jelusic and Zlender 2013). This paper deals only with the limit-equilibrium approach, based on a two-part wedge mechanism.

For any slope, it is possible to identify the critical two-part mechanism, which requires the greatest horizontal reinforcement force. This paper presents the results of a newly developed computer code that is based on limit-equilibrium analyses. This code quantifies earth pressure coefficients to determine the internal design of a reinforced slope and to identify the potential failure surfaces. Dimensionless earth pressure coefficients were calculated for various pore-water pressures within the slope. These calculations can be used to determine the total reinforcement force. The effect of pore-water pressure on potential failure surfaces is also presented in this paper.

Two-Part Wedge Mechanism with Horizontal Reinforcement

Seepage in embankments and slopes is one of the important factors that affect stability, and many slope failures are caused by seepage (Khalilzad et al. 2014). Estimates of pore-water pressure must come from relevant locations in the slope. These pore pressures are

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usually estimated from groundwater conditions, which may be specified by one of the following methods: phreatic surface, piezometric data, piezometric surface, constant pore-water pressure, and pore-water pressure ratio. The pore-pressure ratio provides a simplified but approximate method to characterize seepage. piezometric When the location of the phreatic surface is unknown or unpredictable, it is convenient to assume a pore pressure ratio so that the adverse effect of water can be included in the stability analysis.

The pore-pressure ratio r_u is defined as the ratio between the water pressure and the overburden pressure, or

$$r_u = \frac{u}{\gamma \cdot h} \tag{1}$$

where u is the pore-water pressure; γ is the total unit weight of soil; and h is the depth of the soil between the ground surface and the failure surface (Bishop and Morgenstern 1960).

In the two-part wedge mechanism, the slope is divided into two blocks. The forces acting on the two wedges appear in Fig. 1. Algebraic definitions are defined and used to identify the critical failure surface (Fig. 2). Symbols that are used in the figures are described below. The two-part wedge mechanism has a vertical interwedge boundary. When the interwedge boundary does not have any friction, it provides inherently conservative solutions combined with reasonable simplicity and is particularly suitable for reinforced soil geometries. Therefore, when the interwedge angle of friction is zero, the total quantity of the horizontal reinforcement force required, T_{tot}, is defined with Eq. 2 (Highways Agency 1994)

$$\begin{aligned} T_{\text{tot}} = T_1 + T_2 = {} & \frac{(w_1 + Q_1) \cdot (\tan\theta_1 - \varphi_1) + (U_1 \cdot \tan\varphi_1 - K_1)/\cos\theta_1}{1 + \tan\theta_1 \tan\varphi_1} \\ & + \frac{(w_2 = Q_2) \cdot (\tan\theta_2 - \tan\varphi_2) + (U_2 \cdot \tan\varphi_2 - K_2)/\cos\theta_2}{1 + \tan\theta_2 \tan\varphi_2} (\text{kN/m}) \end{aligned} \tag{2}$$

Once the dimensions of the reinforced slope are defined, the out-of-balance force (T_{tot}) for a single mechanism is calculated. To identify the critical failure surface, the critical mechanism should be found. The critical mechanism will have the largest value of out-of-balance force ($T_{\max}$).

Optimization Model TMAX

In general, when dealing with reinforced slope structures, determining optimization helps to identify the largest value of out-of-balance force ($T_{\max}$). Previous studies used a brute-force method to identify critical failure surface, such as grid mechanisms. For grid mechanisms, it is important to specify a series of uniformly spaced gridlines, along which the program will perform successive calculations to find the critical failure surface at each intersection point on the grid. The brute-force method uses no special information about the function or its derivatives. Therefore, the grid mechanism may miss narrow peaks and function slowly for multidimensional problems. Contrary to engineering practice, where the optimization of the design parameters is determined through iterative successive calculation attempts, this paper instead focuses on the rigorous optimization of the design parameters based on mathematical programing methods.

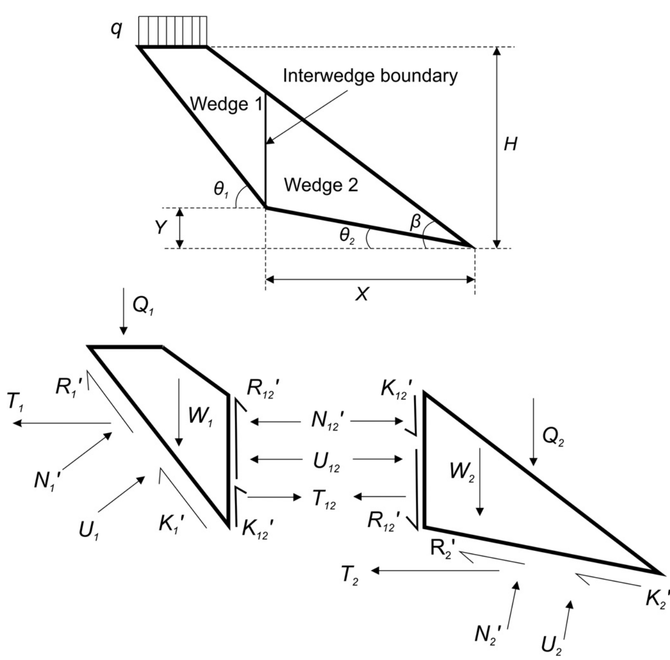

Fig. 1. Two-part wedge failure mechanism

The major advantage of the proposed methodology is that the authors reached the optimal solution after searching a certain number of extreme points with no need to evaluate other extreme points. This proposed methodology increases efficiency because the number of function evaluations needed to obtain the optimal solution is reduced in comparison with the brute-force method.

NLP Problem Formulation

The authors applied a nonlinear programing (NLP) optimization approach. This method was chosen because the problem of slope stability was nonlinear, e.g., the objective function and (in)equality constraints were nonlinear. A general NLP optimization problem/equation can be formulated as follows:

$$\max\ z = f(\boldsymbol{x})$$

subjected to

$$g(\boldsymbol{x}) \le 0$$

$$h(\boldsymbol{x}) = 0$$

$$\boldsymbol{x} \in X = \left\{ \boldsymbol{x} | \boldsymbol{x} \in \boldsymbol{R}^n, \boldsymbol{x}^{\text{Lo}} \le \boldsymbol{x} \le \boldsymbol{x}^{\text{Up}} \right\}$$

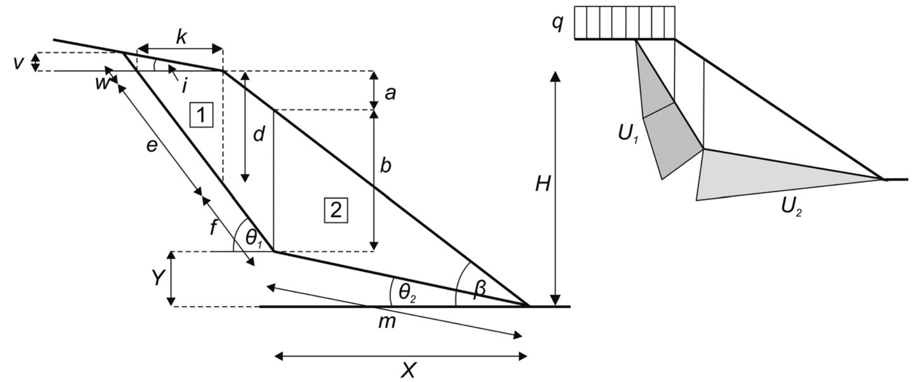

Fig. 2. Definition of two-part wedge geometry and total water pressures acting on the boundaries of each wedge

where $\boldsymbol{x}$ is a vector of the continuous variables defined within the compact set $\boldsymbol{X}$. Functions $f(\boldsymbol{x})$, $g(\boldsymbol{x})$, and $h(\boldsymbol{x})$ are nonlinear functions involved in the objective function z, inequality, and equality constraints, respectively. All functions $f(\boldsymbol{x})$, $g(\boldsymbol{x})$, and $h(\boldsymbol{x})$ must be continuous and differentiable. In the context of slope stability, variables include dimensions, cross-section characteristics, materials, stresses, etc. (In)equality constraints and the bounds of variables represent a rigorous system of loading, resistance, and stress functions taken from slope stability analysis. In this paper, an objective function is proposed to maximize the out-of-balance force $T_{\max}$.

NLP Optimization Model

In accordance with the above NLP problem formulation, an NLP optimization model for a reinforcement slope (TMAX) was developed. Because the model TMAX was developed to be used when the interwedge friction is zero, the results are always conservative. The model optimizes use of the system for various slope angles, structure heights, soil internal friction angles, pore-water pressures, and backfill unit weights. For mathematical modeling and data input/output, the high-level language program *GAMS* was used. The proposed optimization model includes input data, variables, and an out-of-balance force objective function, which is subjected to the structure's constraints.

The following design variables are defined in the optimization model TMAX: the horizontal component of structure X (m), the angle that the base of Wedge 1 makes with the horizontal θ_1 (degrees), and the angle that the base of Wedge 2 makes with the horizontal θ_2 (degrees) (Fig. 1).

The input data (constants) consist of various design data for optimization, i.e., constants/coefficients, which are involved in the objective function and (in)equality constraints. The slope angle β (degrees), the height of structure H (m), the backfill unit weight γ (kN/m^3), the soil internal friction angle φ (degrees), the cohesion c (kN/m^2), the surcharge load q (kN/m^2), and the pore-water pressure ratio r_u comprise the design data.

The objective variable $T_{\max}$ includes the out-of-balance horizontal reinforcement force [Eqs. (2) and (3)]. The various symbols in the objective function have the following meanings (see also Fig. 1): W_i is the weight of Wedge i; N_i' is the force due to effective normal earth pressures acting on the base of Wedge i; U_i is the force due to water pressures acting on the base of Wedge i; R_i' is the force due to effective friction along the base of Wedge i; K_i' is the force due to effective cohesion along the base of Wedge i; N_{12}' is the force due to effective normal earth pressures on the interwedge boundary; U_{12} is the force due to water pore pressures on the interwedge boundary; Q_i is the force due to surcharge load acting on the Wedge i; R_{12}' is the force due to effective friction along the interwedge boundary; T_i is the reinforcement force provided at the base of Wedge i; and T_{12} is the reinforcement force transferred through the interwedge boundary.

Design constraints determine the geosynthetic reinforced structure variables. They are calculated inside their lower and upper bounds. In this way, the horizontal component of structure X (m), the angle that the base of Wedge 1 makes with the horizontal θ_1 (degrees), and the angle that the base of Wedge 2 makes with the horizontal θ_2 (degrees) are bounded by Eqs. (4)–(7).

The objective function

$$T_{\max} = \max\ T_{\text{tot}} \qquad (\text{kN/m}) \tag{3}$$

subjected to design constraints

- Horizontal component of structure X

$$0 \le X \le \frac{H}{\tan\beta} \qquad (\text{m}) \tag{4}$$

- Vertical component of structure Y

$$0 \le Y \le 0.5 \cdot H \qquad (\text{m}) \tag{5}$$

- Angle that the base of Wedge 1 makes with the horizontal θ_1

$$\theta_1 \ge 0 \qquad (\text{degrees}) \tag{6}$$

- Angle that the base of Wedge 2 makes with the horizontal θ_2

$$0 \le \theta_2 \le \beta \qquad (\text{degrees}) \tag{7}$$

and design variables X (m), θ_1 (degrees), and θ_2 (degrees), where

- Forces acting on the two-part wedge mechanism, substituted in the objective function

$$W_1 = \frac{\gamma \cdot [(\mathrm{a}+\mathrm{b})^2 \cdot \cot\theta_1 - a^2 \cdot \cot\beta + v \cdot k]}{2} \qquad (\text{kN/m}) \tag{8}$$

$$W_2 = \frac{\gamma \cdot b \cdot X}{2} \qquad (\text{kN/m}) \tag{9}$$

$$U_1 = \frac{r_u \cdot \gamma \cdot [d \cdot (e+w) + (d+b) \cdot f]}{2} \qquad (\text{kN/m}) \tag{10}$$

$$U_2 = \frac{r_u \cdot \gamma \cdot b \cdot m}{2} \qquad (\mathrm{kN/m}) \tag{11}$$

$$K'_1 = c'_1 \cdot (e + f + w) = c'_1 \cdot (q + w) \qquad (\mathrm{kN/m}) \tag{12}$$

$$K'_2 = c'_2 \cdot m \qquad (\mathrm{kN/m}) \tag{13}$$

$$Q_1 = q \cdot (k + w + \cos\theta_1) \qquad (\mathrm{kN/m}) \tag{14}$$

- Dimensions of the structure, substituted in the forces acting on two-part wedge mechanism

$$a = H - X \cdot \tan\beta \qquad (\mathrm{m}) \tag{15}$$

$$b = (H - Y - a) = X \cdot \tan\beta - Y \qquad (\mathrm{m}) \tag{16}$$

$$d = k \cdot \tan\theta_1 \qquad (\mathrm{m}) \tag{17}$$

$$e = k \cdot \cos\theta_1 \qquad (\mathrm{m}) \tag{18}$$

$$f = \frac{a + b}{\sin\theta_1} - e = g - e \qquad (\mathrm{m}) \tag{19}$$

$$k - (a + b) \cdot \cot\theta_1 - a \cdot \cot\beta = s + t \qquad (\mathrm{m}) \tag{20}$$

$$v = \frac{k}{\cot i - \cot\theta_1}; \text{ if } i = 0, \text{ then } v = 0 \qquad (\mathrm{m}) \tag{21}$$

$$w = \frac{v}{\sin\theta_1} \qquad (\mathrm{m}) \tag{22}$$

- The relation between the out-of-balance horizontal reinforcement force $T_{\max}$ and pressure from the earth

$$K = \frac{T_{\max}}{0.5 \cdot \gamma \cdot H_2} \tag{23}$$

Numerical Example

To interpret the above-proposed optimization approach, this paper presents a numerical example of the NLP optimization of the reinforced slope. The optimization/calculation of the reinforced slope consists of

- The height of structure H (m), the slope angle β (degrees), the backfill unit weight γ (kN/m^3), the soil internal friction angle φ (degrees), the surcharge load q (kN/m^2), and the pore-water pressure ratio r_u were determined. The input data are presented in Table 1 presents the input data.
- The optimization model TMAX was then applied. The following initial values of variables used in the optimizations include $X^L = 0$ m, $\theta_1^L = 30°$, and $\theta_1^L = 30°$. The task of optimizing for the given design parameters focused on finding the maximum out-of-balance force of the reinforced slope $T_{\max}$ (kN/m) as well as the critical failure surface: the horizontal component of slope X (m), the angle that the base of Wedge 1 makes with the horizontal θ_1 (degrees), and the angle that the base of Wedge 2 makes with the horizontal θ_2 (degrees). In addition to this objective, to nondimensionalize the value $T_{\max}$, the parameter K was calculated [Eq. (23)]. *GAMS*/CONOPT2 (generalized reduced-gradient method) was used to solve NLP problems (Drudd 1994). For convenience, a list of $T_{\max}$, K, X/H, Y/H, θ_1, and θ_2 values is provided for the numerical example in Table 2.

To identify the critical failure surface of the reinforced slope, a parametric NLP optimization of the reinforced slope was performed for all 25 combinations of the following different design parameters:

1. Five different slope angles β: 30°, 40°, 50°, 60°, and 70°; and
2. Five different pore-water pressures r_u: 0, 0.1, 0.2, 0.3, and 0.4.

For each combination of the above-listed parameters, an optimization was performed separately using the proposed NLP approach. The above procedure was used to carry out 25 individual NLP optimizations for all 25 combinations. This approach provided 25 different optimal results, i.e., critical failure surfaces (Table 3 presents the results).

Fig. 3 shows a comparison of the dimensionless coefficient K calculated with the NLP optimization model for the reinforcement slope (TMAX) and that obtained using the log spiral failure mechanism, as presented by Jewell (1989). The coefficients obtained using the log spiral failure mechanism were slightly larger than those calculated with the TMAX model.

The effect of the magnitude of the pore-water pressure ratio r_u is demonstrated in Fig. 4, where the curves $r_u = 0$, 0.1, 0.2, 0.3, and 0.4 are given. Fig. 5 illustrates the critical failure surface for the reinforced slope inclined at 70°, assuming the soil internal friction angle equals 20°, and the two values of the pore-water pressure ratio ($r_u = 0$ or 0.4).

An analysis of the optimal results obtained using the NLP for various design parameters led to the following conclusion: the developed TMAX model provided results similar to those published by other authors when using the same conditions, namely those relating to the failure mechanism and the required reinforcement strength. Research has shown only minor differences when using different search algorithms, such as defining the critical mechanism and determining the maximum value of out-of-balance force.

Conclusions

This paper presented a system for identifying failure mechanisms in reinforced slopes. The failure mechanism was performed by use of

Table 1. Input Data for the Numerical Example

Input symbol	Value
H (m)	8
β (degrees)	70
γ (kN/m^3)	20
$\varphi_1 = \varphi_2$ (degrees)	20
$c'_1 = c'_2$ (kN/m^2)	0
q (kN/m^2)	0
r_u	0

Table 2. Results of the Optimization for the Numerical Example

Output symbol	Value
$T_{\max}$ (kN/m)	258.9
K	0.405
X/H	0.231
Y/H	0
X (m)	1.85
θ_1 (degrees)	52.46
θ_2 (degrees)	0

Table 3. Optimal Results Obtained with the NLP for Various Design Parameters

β (degrees)	r_u	$T_{\max}$ (kN/m)	K	X/H	Y/H	X (m)	θ_1 (degrees)	θ_2 (degrees)
70	0	258.9	0.405	0.231	0.000	1.85	52.46	0.00
60	0	227.7	0.356	0.359	0.000	2.87	50.81	0.00
50	0	190.4	0.298	0.503	0.000	4.03	48.61	0.00
40	0	142.5	0.223	0.673	0.000	5.38	45.26	0.00
30	0	76.0	0.119	0.866	0.014	6.93	39.27	0.93
70	0.1	263.5	0.412	0.158	0.000	1.26	48.68	0.00
60	0.1	229.7	0.359	0.272	0.000	2.17	46.56	0.00
50	0.1	192.0	0.300	0.404	0.000	3.23	44.00	0.00
40	0.1	145.6	0.227	0.559	0.000	4.47	40.41	0.00
30	0.1	83.4	0.130	0.710	0.000	5.68	34.43	0.00
70	0.2	280.1	0.438	0.119	0.000	0.96	45.42	0.00
60	0.2	242.1	0.378	0.213	0.000	1.71	42.59	0.00
50	0.2	202.3	0.316	0.327	0.000	2.61	39.64	0.00
40	0.2	155.9	0.244	0.463	0.000	3.70	36.00	0.00
30	0.2	96.3	0.150	0.605	0.000	4.84	30.68	0.00
70	0.3	301.9	0.472	0.096	0.000	0.77	42.58	0.00
60	0.3	260.7	0.407	0.173	0.000	1.39	39.12	0.00
50	0.3	219.0	0.342	0.269	0.000	2.16	35.86	0.00
40	0.3	172.2	0.269	0.389	0.000	3.11	32.25	0.00
30	0.3	113.8	0.178	0.527	0.000	4.21	27.58	0.00
70	0.4	326.8	0.511	0.081	0.000	0.64	39.86	0.00
60	0.4	283.5	0.443	0.145	0.000	1.16	35.94	0.00
50	0.4	240.7	0.376	0.227	0.000	1.81	32.49	0.00
40	0.4	193.6	0.302	0.332	0.000	2.66	28.97	0.00
30	0.4	136.0	0.213	0.466	0.000	3.72	24.83	0.00

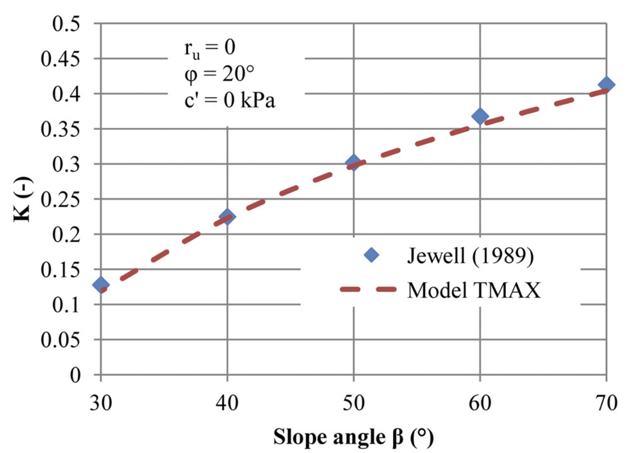

Fig. 3. Comparison of the required strength of reinforcement (expressed by K), obtained using the TMAX model and the log spiral failure mechanism

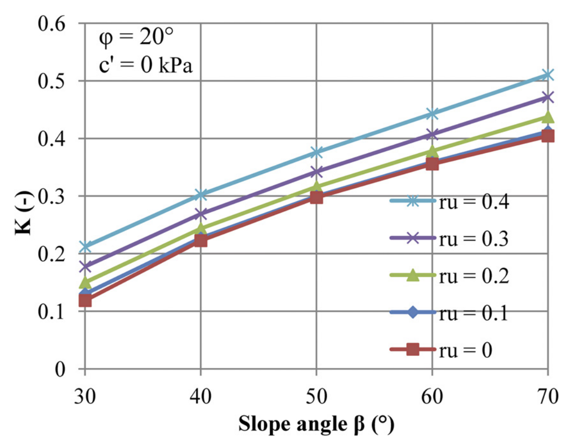

Fig. 4. Effect of pore-water pressure ratio r_u on the required strength of reinforcement (expressed by K)

NLP. For this purpose, a NLP optimization model (TMAX) was developed. The model was based on the objective function, which was subjected to design constraints. As the model was developed in a general form, the optimization of the system can be performed for differing structure geometries, backfill unit weights, soil shear resistances, and pore-water pressures. The optimization was designed to be used for identifying failure mechanisms and finding the required reinforcement strength of a reinforced slope.

The authors recommend that geomechanical problems be solved simultaneously by defining the forces acting on the two-part wedge mechanism and putting them into the optimization model. First, the design parameters have to be determined from the geological conditions of a selected location. Next, the optimization is performed, which provides both the maximum out-of-balance force and the identification of the failure mechanism. Numerical examples that describe the optimization of a reinforced slope were used to interpret the optimized approach. The results show that the increase of pore-water pressure requires an increase of the required strength of the reinforcement, especially when the slope angle is large. The numerical example demonstrates two things: the effect that pore-water pressure has on the required strength of the reinforcement, and the efficiency of the optimization approach introduced in this paper.

The proposed methodology uses an effective, efficient, and reliable algorithm: effective because it calculates the largest value of

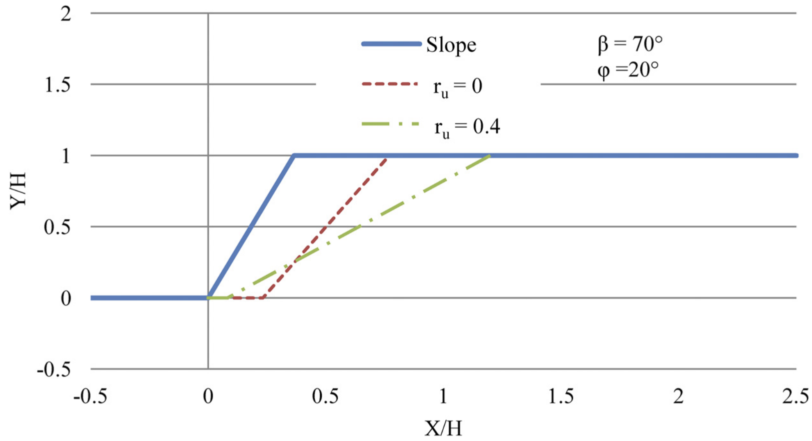

Fig. 5. Critical failure surface for various pore-water pressure ratios

out-of-balance force T_{max} slightly smaller than the log spiral failure mechanism; efficient because the number of function evaluations needed to obtain T_{max} is reduced in comparison with the brute-force method; and reliable (robust) because the algorithms always succeed in finding the T_{max}. Additionally, the nonlinear optimization model shown in this paper can easily be tailored or modified to represent new real situations (e.g. additional external forces) of interest to the users.

References

Abramson, L. W. (2002). *Slope stability and stabilization methods*, Wiley, New York.

Bai, T., Qiu, T., Huang, X., and Li, C. (2014). "Locating global critical slip surface using the Morgenstern-Price method and optimization technique." *Int. J. Geomech.*, 10.1061/(ASCE)GM.1943-5622.0000312, 319–325.

Bathurst, R. J., Miyata, Y., Nernheim, A., and Allen, A. M. (2008). "Refinement of K-stiffness method for geosynthetic reinforced soil walls." *Geosynthetics Int.*, 15(4), 269–295.

Bishop, A. W., and Morgenstern, N. (1960). "Stability coefficients for earth slopes." *Géotechnique*, 10(4), 129–150.

Drudd, A. S. (1994). "CONOPT—a large-scale GRG code." *ORSA J. Comput.*, 6(2), 207–216.

Duncan, J. M. (1996). "State of the art: limit equilibrium and finite-element analysis of slopes." *J. Geotech. Engrg.*, 10.1061/(ASCE)0733-9410(1996)122:7(577), 577–596.

GAMS [Computer software]. Scientific Press, Redwood City, CA.

Highways Agency. (1994). "Design methods for the reinforcement of highway slopes by reinforced soil and soil nailing techniques." *HA 68/94*, Design manual for roads and bridges, Part 4, Highways Agency, London.

Hopkins, T. C., Allen, D. L., and Deen, R. C. (1975). "Effect of water on slope stability." *Res. Rep. 435*, Div. of Research, Kentucky Dept. of Transportation, Frankfort, KY.

Huang, Y. H. (1986). "Unsteady state phreatic surface in earth dams." *J. Geotech. Engrg.*, 10.1061/(ASCE)0733-9410(1986)112:1(93), 93–98.

Huang, Y. H. (2014). *Slope stability analysis by the limit equilibrium method: Fundamentals and methods*, ASCE, Reston, VA.

Jelusic, P., and Zlender, B. (2013). "Soil-nail wall stability analysis using ANFIS." *Acta Geotech. Slov.*, 10(1), 61–73.

Jewell, R. A. (1989). "Revised design charts for steep reinforced slopes." *Reinforced embankments: theory and practice*, Thomas Telford, London, 1–30.

Khalilzad, M., Gabr, M. A., and Hynes, M. E. (2014). "Effects of woody vegetation on seepage-induced deformation and related limit state analysis of levees." *Int. J. Geomech.*, 10.1061/(ASCE)GM.1943-5622.0000304, 302–312.

Leshchinsky, D., and Boedeker, R. H. (1989). "Geosynthetic reinforced soil structures." *J. Geotech. Engrg.*, 10.1061/(ASCE)0733-9410(1989)115:10(1459), 1459–1478.

Liu, F., and Zhao, J. (2013). "Limit analysis of slope stability by rigid finite-element method and linear programming considering rotational failure." *Int. J. Geomech.*, 10.1061/(ASCE)GM.1943-5622.0000283, 827–839.

Schmertmann, G. R., Chouery, V. E., Johnson, R. D., and Bonaparte, R. (1987). "Design charts for geogrid-reinforced soil slopes." *Proc., Geosynthetics '87 Conf.*, Vol. 1, Industrial Fabrics Association International, New Orleans, LA, 108–120.

Vieira, C. S., Lopes, M. L., and Caldeira, L. M. (2011). "Earth pressure coefficients for design of geosynthetic reinforced soil structures." *Geotext. Geomembr.*, 29(5), 491–501.

Wright, S. G., and Duncan, J. M. (1991). "Limit-equilibrium stability analyses for reinforced slopes." *TRB Paper No. 910441*, Transportation Research Board, Washington, DC.

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AQ7: Is CONOPT2 a computer program? If so, please add a reference to the list with the manufacturer and its location.

AQ8: Please provide page numbers for Jewell (1989).

AQ9: Please provide the name of the publisher and its location (city and state/country) for Schmertmann et al. (1987).