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Constitutive Model and Failure Criterion Analysis of High-Density Polyethylene Pipe Under Thermal-Mechanical Load

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ABSTRACT

As a typical viscoelastic material, high-density polyethylene (HDPE) properties are sensitive to service temperature and strain rate. In order to establish a new constitutive model and failure criterion equation for HDPE pipes at different service temperatures and strain rates, tensile tests of HDPE pipes at different temperatures and strain rates were carried out, and then a constitutive model of PE pipe considering the influence of temperature and strain rate was proposed by modifying the Suleiman's hyperbolic constitutive equation. Finally, a ductile failure criterion of HDPE material was established based on the strength theory.

Keywords: High density polyethylene; buried pipeline; thermal-mechanical load; failure criterion; constitutive model.

1. INTRODUCTION

Due to advantageous engineering properties of reduced installation costs, light-weight, high ductility, corrosive resistance, excellent long-term performance as pressure pipes, high-density polyethylene (HDPE) materials are widely used for natural gas, sewerage networks, water transport structures, cooling units of thermal and nuclear power plants, etc. As a type of viscous-elastic material, the mechanical properties and damage evolution of HDPE material are strain rate-dependent, temperature-dependent and time-dependent. A wide range of studies on HDPE pipelines have been investigated on frost action, land subsidence,

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ground occupation, seismic waves or synergistic effects of two or more factors.

In the study conducted by Gulen M[1]the horizontal stresses at the springline and the lateral earth pressure coefficient was considered as functions of the horizontal de-flection of the HDPE pipe. Experimental and Finite element studies performed by Wu et al [2] proposed a damage prediction mode and critical curves at different damage levels under surface blast loading of HDPE pipe. An et al[3] discussed the mechanical behavior of buried HDPE pipelines under blasting demolition of engineered buildings. The research results show that the top, bottom and the arches of the buried HDPE pipeline are the stress concentration areas, and the overlying pavement structure is more important for reliability. Alin et al[4] analyzed the mechanical behavior of HDPE pipes under accidental excavation loads by means of numerical and experimental investigation, and the results show that specific deformation must be considered for the design criterion. Shi et al[5] investigated the stress distribution of HDPE pipe in the pipe gallery used in nuclear power plants. The results show that stresses concentrate in the fusion regions along the inner surface of mitered elbows. Zhang et al[6] investigated the mechanical behaviors and failure mechanism of HDPE bellows using full-scale blasting tests. The research results show that HDPE pipes are more prone to be destroyed under circumferential compression. Zahedi[7] found that, at the same operating condition, the maximum von Mises stress value in the defective buried polyethylene gas pipeline is significantly higher than that in the normal pipe.

For complicated service environments of buried HDPE pipes during the practical application, more and more researchers pay attention to damage or failure behavior considering two or more damage factors. Li et al[8]proposed a hyper-elastoplastic constitutive model that established the isotropic plasticity-ductility damage correction under tensile and bending conditions. By considering two factors of relative density and different thermal conditions, Necmettin et al[9] found that the deformation of the buried HDPE pipes increases with an increase in relative density, and the maximum service temperature is 50 ℃ for the appearance of the maximum displacement and bending moment values. Wu et al[10-11]investigated the stress and deformation behavior of HDPE pipe under the action of the scratch defect and land subsidence, and the ultimate bearing capacity and its laws of the pipeline under different parameters were proposed.

However, the literatures discussed above were not interpreted together in terms of thermal-mechanical perspective within the scope of environmental temperature and external load. Considering these factors separately for giving predictable mechanical properties or residual life is traditional and conservative. Evaluating all factors together is crucial in terms of safety, cost, and long service life. By modifying Suleiman’s hyperbolic constitutive equation, the main aims of the present paper were to establish a general stress-strain equation of HDPE pipes and ductile failure criterion under the synergistic effect of temperature and external load, to reflect the real service conditions and provide a good understanding of damage condition and failure mode.

2. MATERIAL AND EXPERIMENT

2.1 Material preparation

A HDPE pipe with an outer diameter of 110mm and a wall thickness of 10mm was used in the present paper. All tensile samples were cut along the axial direction of the same pipeline, and then, polished measurement with 400# SiC sandpaper was car-ried out to avoid the influence of surface factors on the test, such as scratches. For each test, five specimens were prepared to reduce the test error.

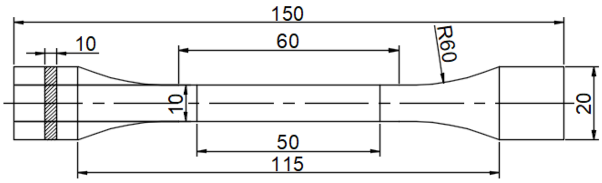

Fig.1 Morphology and size of specimen

2.2 Experimental procedure

The mechanical properties of polyethylene pipelines are greatly affected by the service temperature. To eliminate the influence of the internal and external tempera-ture difference of the sample on the test results, the holding time-independent verification test was carried out. The operating temperature of specimens was set as 273.15K and held for 1h, 2hs and 3hs respectively, and the tensile rate was set to 0.01104/s. The stress-strain curve is shown in Fig.2(a) and the standard deviations value are shown in Fig.2(b)

As seen in Fig. 2, less difference was shown on the engineering stress-strain curves of the samples under different holding times. Yield strength values are 33.9MPa, 33.4MPa and 33.8MPa respectively, and elastic modulus values and yield stress values are almost as same as each other. In the present paper, elastic modulus and yield stress are only paid attention to, consequently, 1h was selected as the holding time for the tensile tests.

Generally, the service temperature range of the PE buried pipeline is 253.15K ~ 311K, different temperature values of 258.15K, 273.15K, 288.15Kand 303.15K were selected for the test, and strain rates of 0.01104/s, 0.00219/s, 0.00087/s and 0.00022/s were selected.

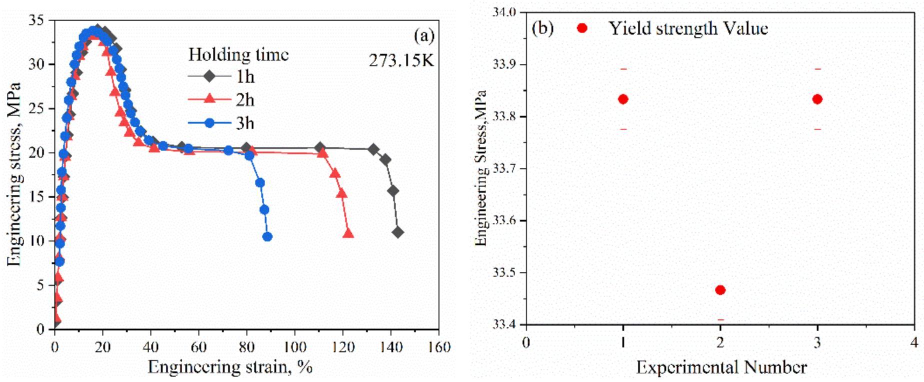

Fig.2 The engineering stress-strain curves of HDPE pipelines under different holding times.

In this paper, the stress-strain curves up to the yield strength were only focused on, the curve presented is sufficient to achieve the engineering strain value of 0.2. To re-duce the error caused by sample processing, tensile test equipment and human factors, three samples are stretched at each temperature and strain rate.

3. RESULTS AND DISCUSSION

3.1 Discussion of Experimental results

The engineering stress-strain curves under different temperatures and strain rates are shown in Fig. 3(a)~ Fig. 3(d). As seen in Fig. 3, at the same operating temperature, the elastic modulus and yield stress of the PE pipe increase with the strain rate in-creases. However, at the same strain rates, the elastic modulus and yield stress de-crease with test temperature increasing. Based on the above analysis, it can be clearly seen that the mechanical properties of PE pipe are highly temperature-dependent and strain rate-dependent. These conditions mean that the yield stress changes when the service temperature or external load of the pipe changes, which reveals that the failure criterion based on the ultimate load also changes.

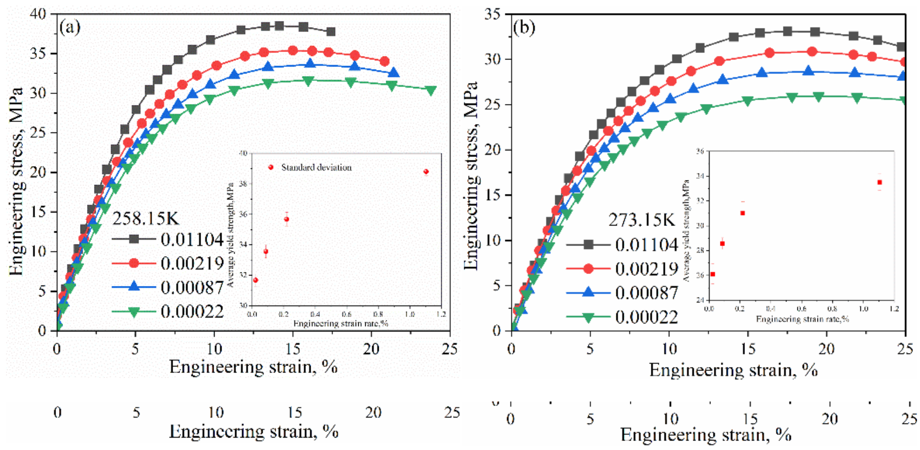

Fig.3 Engineering stress-strain curves of HDPE pipes at different temperatures and different strain rates

3.2 Modification of the Suleiman constitutive model of HDPE material

It is as well-known that the mechanical properties of HDPE pipes are highly temperature-dependent and strain rate-dependent, but now there is no general constitutive model that can be used for describing the mechanical variation of PE pipes under the synergistic effect of temperature and external load.

Due to the convenience of data acquisition, the Suleiman hyperbolic model is one of the main constitutive models for describing the stress-strain relationship. On one hand, stress-strain data of tensile tests at different temperatures and different strain rates only be needed, and then the relevant parameters can be obtained by simple fitting. On the other hand, the Suleiman hyperbolic model is accorded with the actual situation while considering the rate correlation of HDPE materials at a specific ambient temperature.

In this paper, modified research work for the Suleiman hyperbolic model has been carried out, and then the modified Suleiman constitutive equation considering the synergistic effect of temperature and load is proposed. The Suleiman constitutive mod-el is shown in Eq. (1)

$$\sigma_{\mathrm{T}} = \frac{\varepsilon_{\mathrm{T}}}{a + b\varepsilon_{\mathrm{T}}} \tag{1}$$

Where σ_{T} is the true stress, MPa; ε_{T} is the true strain, dimensionless; a and b are the parameters related to the initial modulus and yield strength. As seen from Eq. (1), it demonstrates the general form of the nonlinear relationship. Values of a and b could be estimated by a nonlinear regression analysis based on the true stress-strain curves.

Since the stress-strain curves obtained by tensile test are engineering stress-engineering strain curves, it is necessary to convert the engineering stress-strain value into the real stress-strain values, by determining the Poisson’s ratio value as 0.45[12]. The conversion formulas of engineering stress-strain and real stress-strain are shown in the following Eq. (2) and Eq. (3).

$$\varepsilon_{\mathrm{T}} = \ln(1 + \varepsilon_{\mathrm{E}}) \tag{2}$$

$$\sigma_{\mathrm{T}} = \frac{\sigma_{\mathrm{E}}}{(1 - \mu\varepsilon_{\mathrm{E}})^2} \tag{3}$$

Where ε_{E} is engineering strain, dimensionless; σ_{E} is engineering stress, MPa; μ is Poisson’s ratio, dimensionless. The real stress-strain curves transformed from the engineering stress-strain curves are shown in Fig. 4. By this transformation, a represents the intercept and b represents the slope of the strain/stress ratio ($\varepsilon_{\mathrm{T}} / \sigma_{\mathrm{T}}$)versus strain relationship, as shown in Eq. (4).

$$\frac{\varepsilon_{\mathrm{T}}}{\sigma_{\mathrm{T}}} = a + b\varepsilon_{\mathrm{T}} \tag{4}$$

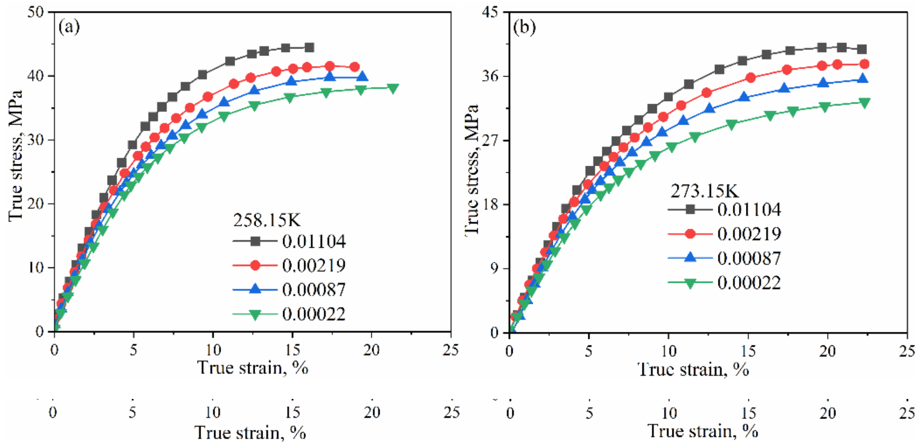

Fig.4 The true stress-strain curves of HDPE material under different strain rates and different temperatures

According to Eq. (4), symbols *a* and b are obtained by linear regression of the ratio of real strain/stress ratio ($\varepsilon_{\mathrm{T}} / \sigma_{\mathrm{T}}$) versus real strain. The results of the regressions are given in Table 1. And**3.2.2.Strain of the middle cavity section**

the degree of linearity is clearly reflected in the reported coefficient of determination R^2 values. According to the correlation coefficient R^2 in the table, the fitting results are in accordance with the experimental data.

Table 1 Regression results of a and b at different strain rates and different temperatures

Temperature（K）	Strain rate, s^{-1}	Constant, *a*	Constant, *b*	Correlation coefficient, R^2
258.15K	0.01104	0.0008294	0.01658	0.9927
	0.00219	0.0010000	0.01776	0.9888
	0.00087	0.0010852	0.01855	0.9914
	0.00022	0.0013000	0.01948	0.9894
273.15K	0.01104	0.0010600	0.01934	0.9939
	0.00219	0.0013000	0.02001	0.9945
	0.00087	0.0015000	0.02062	0.9942
	0.00022	0.0017000	0.02280	0.9963
288.15K	0.01104	0.0012800	0.02176	0.9974
	0.00219	0.0014794	0.02389	0.9989
	0.00087	0.0017000	0.02513	0.9988
	0.00022	0.0020200	0.02620	0.9995
303.15K	0.01104	0.0018000	0.02711	0.9984
	0.00219	0.0021000	0.02895	0.9978
	0.00087	0.0022653	0.03134	0.9998
	0.00022	0.0028879	0.03290	0.9993

However, in order to propose a constitutive equation for HDPE material, it is necessary to obtain the relationship between constants (a and b) and the strain rate. And this approach can be undertaken by performing separate logarithm regressions of constants (a and b) and the strain rate. The results as given in Fig.5, and relationship expression equations are shown in Eq.(5) and Eq.(6).

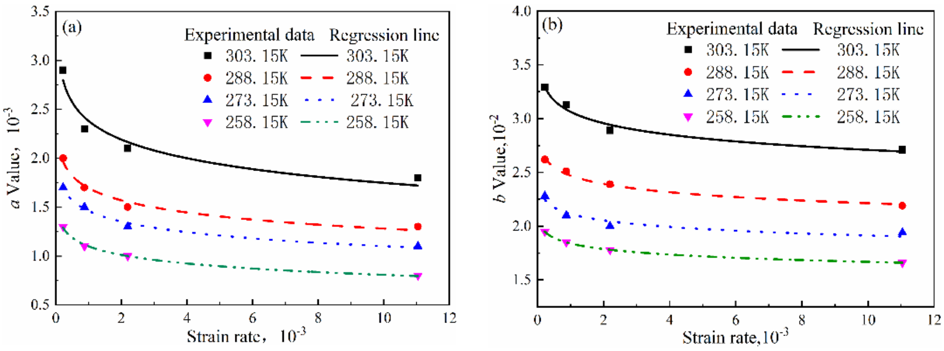

Fig.5 The relationship between constants (a and b) and strain rate at different temperatures

$$a = A_1 + A_2 \cdot \ln(\dot{\varepsilon}) \tag{5}$$

$$b = B_1 + B_2 \cdot \ln(\dot{\varepsilon}) \tag{6}$$

Where A_1, A_2, B_1 and B_2 are fitted parameters, $\dot{\varepsilon}$ is strain rate. And values of A_1, A_2, B_1 and B_2 at different temperatures are shown in Table 2.

Table 2 The values of parameters A1, A2, B1 and B2 at different temperatures

Temperature, K	A_1	A_2	R^2	B_1	B_2	R^2
258.15K	-0.000126442	0.000224995	0.99553	-0.000741696	0.01326	0.99995
273.15K	-0.000156286	0.000380274	0.99512	-0.000869045	0.01513	0.9221
288.15K	-0.000179687	0.000452588	0.99778	-0.00111	0.01701	0.98293
303.15K	-0.000274789	0.000482072	0.99541	-0.00154	0.02001	0.96099

The constitutive model of HDPE material associated with strain rate are shown as the following Eq. (7) by substituting Eq. (5) and Eq. (6) into Eq. (1).

$$\sigma = \frac{\varepsilon}{[A_1 + A_2 \ln(\dot{\varepsilon})] + [B_1 + B_2 \ln(\dot{\varepsilon})] \cdot \varepsilon} \tag{7}$$

For temperature-dependent characteristics of HDPE material, it is necessary to obtain the relationship between fitted parameters (A_1, A_2, B_1 and B_2) and temperature. The results are shown in Fig. (6) by way of exponential regression. The equations expressing the relationship of parameters (A_1, A_2, B_1 and B_2) versus temperature are shown in Eq. (8) to Eq. (11).

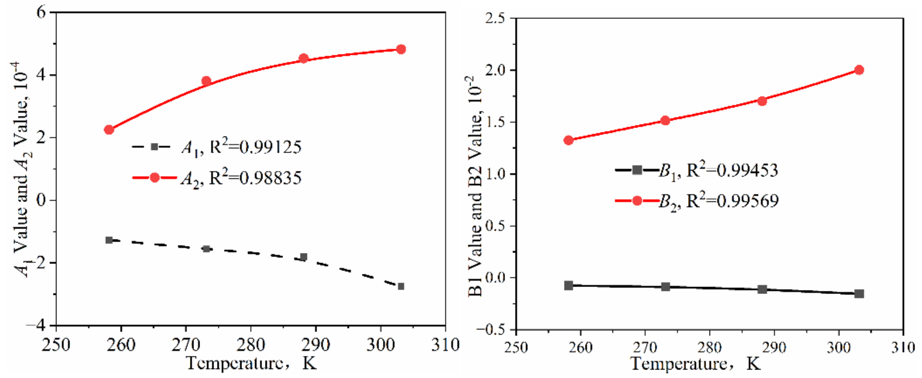

Fig.6 The relationship between parameters (A1, A2, B1 and B2) and temperature

$$A_1 = m_1 \cdot e^{n_1 \cdot T} \tag{8}$$

$$A_2 = m_2 \cdot e^{n_2 \cdot T} \tag{9}$$

$$B_1 = k_1 \cdot e^{l_1 \cdot T} \tag{10}$$

$$B_2 = k_2 \cdot e^{l_2 \cdot T} \tag{11}$$

Where symbols m_1、m_2、n_1、n_2、k_1、k_2、l_1 and l_2 are fitted parameters, whose values are shown in table 3.*T* is temperature, ℃。

Table 3 The values of parameters m1, m2, n1, n2, k1, k2, l1 and l2

Parameter	m_1	n_1	m_2	n_2	k_1	l_1	k_2	l_2
Value	-1.59229 $\times10^{-4}$	1.728 $\times10^{-2}$	2.99187 $\times10^{-4}$	1.627 $\times10^{-2}$	-9.11054 $\times10^{-4}$	1.698 $\times10^{-2}$	1.499 $\times10^{-2}$	9.230 $\times10^{-3}$

Constitutive model of HDPE material assisted with strain rate and temperature is shown as the following Eq. (12) by substituting Eq. (8) ~Eq. (11) into Eq. (7).

$$\sigma = \frac{\varepsilon}{[-1.59229\times10^{-4}e^{0.01728T}+2.99187\times10^{-4}e^{0.01627T}\ln(\dot{\varepsilon})]+[-9.11054\times10^{-4}e^{0.01698T}+0.01499e^{0.00923T}\ln(\dot{\varepsilon})]\cdot\varepsilon} \tag{12}$$

In the foregoing analysis, Eq. (12) is the general modified Suleiman constitutive model for HDPE material combining actions of external load with temperature. The constitutive model at any external load and any temperature could be proposed if the service temperature and strain rate is given.

3.3 Failure criterion of PE material

As one of the main failure types of HDPE material, ductile failure behavior has been paid more attention in recent years. It is safe to take the yield strength as the failure criterion of HDPE pipe [13]. In this paper, based on the fourth strength theory, shown as von-mises yield failure criterion, the yield stress of HDPE pipe was adopted as the failure criterion.

In the foregoing analysis, as a semi-crystalline polymer material, the yield stress of HDPE material is temperature-dependent and strain rate-dependent. Generally, Semi-crystalline polymer material has an interpenetrating network structure, which is composed of lamellar crystalline regions and entangled amorphous molecular chains [14]. The reason of the deformation of the crystal region and the amorphous region is different. Deformation of the former is the mutual slip behavior of the wafer, and the reason for the latter is the entanglement molecular chain unwinding behavior. Both of them conform to the theory of flow mechanics. Consequently, the relationship between the yield stress and strain rate of the pipe could be described using the Eyring flow theory model [15]. Moreover, by processing the tensile data of PE samples at different temperatures and different strain rates, it is found that the yield stress of pipes at different temperatures has a logarithmic relationship with the strain rate as shown in Fig.7, and the relationship expression is shown as Eq. (13).

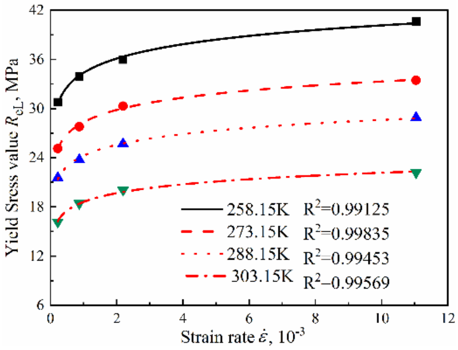

Fig.7 The relationship between yield stress and strain rate of pipes at different operating temperatures

$$R_{eL} = F + Q \cdot \ln \dot{\varepsilon} \tag{13}$$

Where R_{eL} is yield stress, MPa; F and Q are fitting parameters, and values of F and Q at different temperatures are shown in Table 4.

Table 4 The values of parameters F and Q

Temperature, K	258.15K	273.15K	288.15K	303.15K
F	51.73798	43.23347	37.33419	29.27539
Q	2.51591	2.15651	1.89369	1.544

For the yield stress of HDPE material is temperature-dependent, based on Eq. (13), exponential regression is carried out for parameters of F and Q. The result is shown in Fig.8, and the relationship expressions are shown in Eq. (14) and Eq. (15).

$$F = f_1 \cdot e^{f_2 \cdot T} \tag{14}$$

$$Q = q_1 \cdot e^{q_2 \cdot T} \tag{15}$$

Where, f_1、f_2、q_1 and q_2 are fitting parameters; f_1、f_2、q_1 and q_2 values are shown in Table 5.

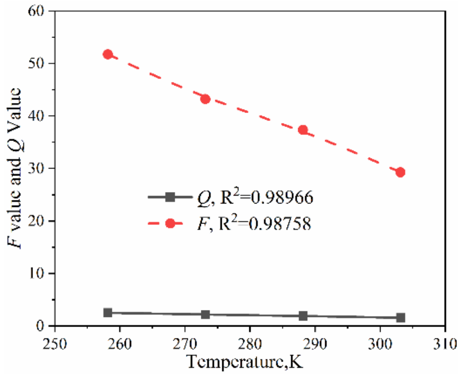

Fig.8 The relationship between parameters F and Q versus temperature

Table 5 The values of parameters f1, f2 , q1 and q2

f_1	f_2	q_1	q_2
43.34053	-0.01208	2.15977	-0.01043

Failure criterion expression equation of HDPE material, expressed as yield stress, is shown as the following Eq. (16) by substituting Eq. (14) and Eq. (15) into Eq. (13).

$$R_{eL} = 43.3849 \cdot e^{-0.0124 \cdot T} + 2.1615 \cdot e^{-0.0106 \cdot T} \cdot \ln \dot{\varepsilon} \tag{16}$$

Based on the above analysis, Eq.(13) is the general yield stress equation for HDPE material combining actions of external load and temperature. The yield stress value at any external load and any temperature could be proposed if the service temperature and strain rate is given. In practical engineering application, considering the application of safety factor, the failure criterion of HDPE pipeline under the synergistic effect of temperature and external load is expressed as following Eq.(17).

$$[\sigma] = \left(43.3849 \cdot e^{-0.0124 \cdot T} + 2.1615 \cdot e^{-0.0106 \cdot T} \cdot \ln \dot{\varepsilon}\right) / n_s \tag{17}$$

Where n_s is safety factor.

4. CONCLUSION

Due to the mechanical properties of HDPE material being strain rate-dependent and temperature-dependent, there is no general constitutive model that could be used for describing the mechanical behavior at conditions of different temperatures and different external loads.

(1) Combing actions of temperature at a different external load, a new constitutive model for describing the mechanical behavior is proposed by modifying the Suleiman constitutive model first

(2) The ductile failure criterion of HDPE material is established based on the strength theory, and the failure criterion equation at conditions of different temperatures and external loads

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REFERENCES

[1] Gulen Murat,Kilic Havvanur. Effects of Pipe Deflection and Arching on Stress Distribution and Lateral Earth Pressure Coefficient in Buried Flexible Pipes. Applied sciences. 2024. 14(4): 1667.

[2] Wu Tingyao, Jiang Nan,Zhou Chuanbo, et al. Experimental and numerical investigations on damage assessment of high-density polyethylene pipe subjected to blast loads. Engineering failure analysis. 2022. 131: 105856.

[3] An Zhaotun, Tang Qiao,Li Hu, et al. Experimental and numerical study of shallow‐buried high‐density polyethylene pipeline under collapse touchdown impact. Energy science & engineering. 2022. 10(2): 355-373.

[4] Dinita Alin, Ramadan Ibrahim,Tanase Maria. Experimental and Numerical Study Regarding the Behavior of HDPE Pipes under Quasi-Static Point Loads. Journal of Pipeline System Engineering and Practice. 2023. 14(1): 04022072.

[5] Shi Jianfeng, Hu Anqi, Yu Fa, et al. Finite element analysis of high-density polyethylene pipe in pipe gallery of nuclear power plants. Nuclear Engineering and Technology. 2021. 53(3): 1004-1012.

[6] Zhang Yuqi, Jiang Nan,Zhou Chuanbo, et al. Mechanical behaviors and failure mechanism of HDPE corrugated pipeline subjected to blasting seismic wave. Alexandria engineering journal. 2023. 67: 597-607.

[7] Khademi-Zahedi R. Application of the finite element method for evaluating the stress distribution in buried damaged polyethylene gas pipes. Underground Space. 2019. 4(1): 59-71.

[8] Li Xiao, Zhao Bo,Yu Yuxin, et al. Investigation on Constitutive and Ductile Damage Mechanism of High-Density Polyethylene under Tensile and Bending Conditions. Journal of Pipeline System Engineering and Practice. 2023. 14(2): 4022075.

[9] Polat Necmettin, Erenson Can,Uğur Terzi Niyazi. The deformation characteristics of

buried HDPE transfer lines in trenches under the effect of temperature. Revista de la construcción (Universidad Católica de Chile). 2021. 20(3): 452-462.

[10] Wu Ying, You Xiao,Zha Sixi. Investigation of mechanical behavior of buried DN110 polyethylene pipe with a scratch defect under land subsidence. Engineering failure analysis. 2021. 125: 105371.

[11] Wu Ying, You Xiao, Zha Sixi. Mechanical behavior analysis of buried polyethylene pipe under land subsidence. Engineering failure analysis. 2020. 108: 104351.

[12] Luo Xiangpeng, Lu Shunli, Shi Jianfeng, et al. Numerical simulation of strength failure of buried polyethylene pipe under foundation settlement. Engineering Failure Analysis. 2015. 48: 144-152.

[13] Xu Cheng, Xu Ping, Shi Jian-feng, et al. Prediction of Ductile Rupture Failure Life of HDPE Pipe[J]. Pressure Vessel Technology. 2012, 29(1):1-6.

[14] Hillmansen S, Hobeika S, Haward R N, et al. The effect of strain rate, temperature, and molecular mass on the tensile deformation of polyethylene[J]. Polymer Engineering & Science, 2000, 40(02): 481-489.

[15] Jiang Zhiyong. Structural Evolution of Tensile-Deformed High-Density Polyethylene Investigated by Synchrotron Small-Angle X-ray Scattering[D]. Changchun: Changchun Institute of Applied Chemistry Chinese Academy of Science. (In Chinese)