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Evaluation of the Hydraulic Conductivity of Geosynthetic Clay Liners

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Abstract The hydraulic conductivity of geosynthetic clay liners (GCLs) is not fully understood and certain gaps in knowledge are still present, such as the effect of coupled mechanical and chemical processes. The current study aimed to develop a simplified mathematical model to predict the hydraulic conductivity of GCLs, particularly regarding the coupled effects of mechanical and chemical processes. Based on Darcy's Law and Poiseuille's Law, the method combines diffuse double layer (DDL) theory and fractal theory. External factors such as confining pressure and the concentration of the permeating solution, and inherent properties such as exchangeable cations, ionic radius, montmorillonite surface fractal dimension, the distance between two montmorillonite layers (m) after swelling at the exchangeable cation *i* (*i* denotes the primary exchangeable cations, such as Na^+, Ca^{2+}, K^+, and Mg^{2+} in bentonite), density, and coefficient of viscosity of interlayer water between two montmorillonite layers, were considered. The proposed theoretical model gave relatively accurate predictions. A practical estimate of GCL hydraulic conductivity was also derived. The predictions were compared with experimental results and good qualitative agreement was found. From the experimental results, the proposed prediction model has a maximum deviation of ~1:10–10:1, and the empirical model has a mean deviation of ~1:15–15:1.

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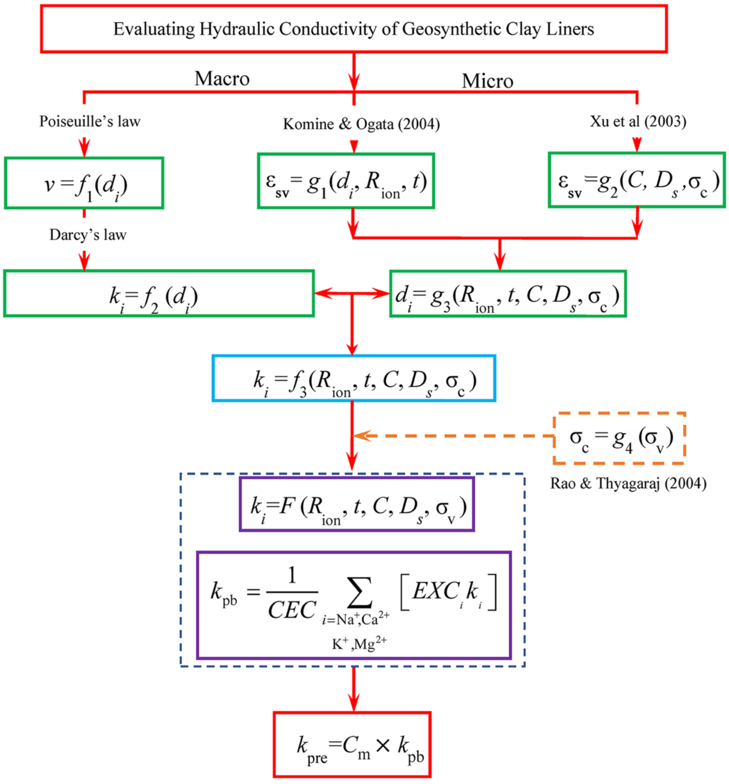
Introduction

In recent years, geosynthetic clay liners (GCLs) have received much attention as barrier systems in municipal solid-waste landfill applications because of their good physical and chemical properties,



Notation:

v: the average velocity

$\boldsymbol{d_i}$: the half distance between two montmorillonite layers after swelling at the exchangeable cation (m)

ε_{sv}: the percentage volume increase of swelling deformation of montmorillonite

$\boldsymbol{R_{ion}}$: the ionic radius

t: the thickness of the montmorillonite layer (m)

C: the Hausdorff measurement of the fractal surface

$\boldsymbol{D_s}$: the surface fractal dimension of montmorillonite in bentonite

$\boldsymbol{\sigma_c}$: the total net vertical stress

$\boldsymbol{k_i}$: the hydraulic conductivity of GCLs (m/sec)

$\boldsymbol{\sigma_v}$: the vertical confining stress

$\boldsymbol{k_{pb}}$: the hydraulic conductivity of bentonite

CEC: the bentonite cation exchange capacity (meq/g)

$\boldsymbol{EXC_i}$: the i-cation exchange capacity (meq/g), i=Na^+, Ca^{2+}, K^+, and Mg^{2+}

$\boldsymbol{k_{pre}}$: the hydraulic conductivity of GCLs

$\boldsymbol{C_m}$: the montmorillonite content

Fig. 1 The flow chart of the theoretical model ability to self-repair, easy processing, and environmental endurance. Extensive experimental studies have investigated the hydraulic conductivity of GCLs (Abuel-Naga et al., 2013; Bouazza, 2002; Bouazza et al., 2006; Giroud et al., 1997; Liu et al., 2014; Rouf et al., 2016; Rowe, 1998; Xue et al., 2012). Numerous investigators have focused on the factors influencing GCL hydraulic conductivity through compatibility tests (Jo et al., 2001; Vasko et al., 2001; Jo et al., 2005; Fox and Ross, 2011; Benson, 2013; Shen et al., 2015). Considering actual field conditions, the coupling effect of the overlying load pressure and the leachate on the hydraulic conductivity of GCLs has been addressed recently (Shackelford et al., 2010; Kang & Shackelford, 2011; Zhu et al., 2015; Malusis et al., 2015; Chen et al., 2016; Scalia et al., 2018, Wang et al., 2019a, b). Naka et al. (2016) presented a state-of-the-art review of the factors impacting the hydraulic conductivity of GCLs, concluding that bentonite type, prehydration, confining pressure, pH, metal concentration, and metal ion type are the strongest influences. Numerical simulations and theoretical models, which have made possible the measurement of hydraulic conductivity of GCLs from information about the porous media structure, are powerful alternatives in predicting this physical parameter (Dexter & Richard, 2009; Guan et al., 2014; Nakano & Miyazaki, 2005; Quinton et al., 2009; Schaap & Leij, 1998; Xie et al., 2018).

Compared to the extensive experimental studies, however, numerical simulations have been relatively few (Bolt, 1956; Bouazza et al., 2013, 2014; Saidi et al., 2006; Siemens et al., 2012; Stępniewski et al., 2011). Theories based on case studies remain quite rare (Benson et al., 2018; Kolstad et al., 2004, 2006; Siddiqua et al., 2011; Yan et al., 2020). Chai and Shen (2018) used diffuse double layer (DDL) theory to analyze the swelling behavior of a Na^+ bentonite used in GCLs with lower dry unit weights and found linear relationships between the calculated double layer thickness and the measured corresponding free swelling index and liquid limit. Dominijanni et al. (2012, 2018) proposed a physical approach to interpreting the phenomenological parameters obtained from laboratory tests. Michels et al. (2019) inferred mesoporous humidity from a space-resolved measurement with a fractal diffusion equation and showed that water transport through a system of clay minerals could be hysteretic. However, they suggested that sample preparation history in Na bentonite has little effect on the water vapor transport through the mesopores. Using the study of Chung and Daniel (2008), Liu et al. (2018) calculated the hydraulic conductivity of GCLs using DDL theory. However, Sposito (1984) stated that the DDL theory cannot predict the dissolved divalent cations accurately, especially in the initial stages of swelling, as was verified by Schanz and Tripathy (2009). Meanwhile, the fractal model is characterized mainly by the fractal dimension, which is affected by the physical properties of porous media. Some studies showed that the fractal theory can be used to determine the swelling properties of bentonite (Boadu, 2000; Thevanayagam & Nesarajah, 1998), especially in the initial swelling stage. Peng et al. (2020) employed small-angle X-ray scattering (SAXS) and liquid nitrogen adsorption (Frenkel-Halsey-Hill (FHH) and Neimark thermodynamic method) to determine the fractal dimension of four Chinese bentonites. The swelling strain and the clay particle thickness changed non-linearly as a function of water content and cation type (Altoé et al., 2016; Michels et al., 2020). Further studies have extended fractal theory in studying bentonite (Li & Xu, 2019; Xiang et al., 2019; Xu et al., 2003, 2004).

The overall objective of the current study was to present a theoretical mathematical equation for calculating the hydraulic conductivity of GCLs, particularly regarding the coupled effect of mechanical and chemical processes. The DDL theory and a new fractal model of montmorillonite were combined. Also investigated was the influence on the hydraulic conductivity of GCLs of confining pressure, the concentration of permeate salt solutions, exchangeable cations, bentonite ionic radius, the surface fractal dimension of montmorillonite, and the distance, density, and coefficient of viscosity of interlayer water between two montmorillonite layers. The flow chart showing how the theoretical model is developed is shown in Fig. 1.

As shown in the flow chart (Fig. 1), from a macro view, the hydraulic conductivity of GCLs can be calculated using Poiseuille's Law and Darcy's Law. From the micro view, the hydraulic conductivity of GCLs can be calculated using the capillary rise concept of clay swelling and fractal dimensions. Combining the micro and macro views, a theoretical model for predicting the hydraulic conductivity of GCL is developed. The effect of confining pressure, the concentration of the permeating solution, the exchangeable cations, the ionic radii of cations, the montmorillonite surface fractal dimension, and the distance between two montmorillonite layers (m) after swelling are considered in this theoretical model.

A Theoretical Model of GCL Hydraulic Conductivity

GCLs consist generally of bentonite sandwiched between two geotextile layers that are needled or sewn together to provide shear strength. Bentonite materials, the most important components of the GCLs, are characterized by potentials for high water retention and swelling (Rowe, 2014). For the bentonite material, Komine (2005) assumed that water goes mainly through two montmorillonite layers swollen by adsorbed water, based on their experimental work (Komine & Ogata, 1996, 2003, 2004; Komine, 2004a, b). Therefore, according to the plane Poiseuille flow equation, the maximum velocity, $v_{\max}$, can be expressed as:

$$v_{\max} = -\frac{(2s_i)^2}{8\mu_{\mathrm{wi}}}\frac{dp}{dx} = -\frac{\rho g(2s_i)^2}{8\mu_{\mathrm{wi}}}\frac{dh}{dx} = -\frac{r_{\mathrm{p}}(2s_i)^2}{8\mu_{\mathrm{wi}}}\frac{dh}{dx} \quad (1)$$

where s_{i} is the half distance between two montmorillonite layers (m) after swelling at the exchangeable cation i (i denotes the primary exchangeable cations, such as Na^+, Ca^{2+}, K^+, and Mg^{2+} in bentonite) (Komine & Ogata, 2003), μ_{wi} is the coefficient of viscosity of interlayer water between two montmorillonite parallel layers (Pa·s), p is hydraulic pressure, x is the coordinate along the flow, $\frac{dp}{dx}$ is differentiation of p with x, ρ is solution density, g is gravitational acceleration, $\frac{dh}{dx}$ is the hydraulic gradient, and r_{p} is the density of interlayer fluid between two montmorillonite parallel-plate layers (Pa/m).

The average velocity v then can be expressed as (Komine & Ogata, 2003):

$$v = \frac{2}{3}v_{\max} = -\frac{2}{3}\times\frac{r_{\mathrm{p}}(2s_i)^2}{8\mu_{\mathrm{wi}}}\frac{dh}{dx} = -\frac{r_{\mathrm{p}}(2s_i)^2}{12\mu_{\mathrm{wi}}}\frac{dh}{dx} \quad (2)$$

Using Darcy's law for flow through a porous medium, the average velocity can be expressed as follows:

$$v = -k\frac{dh}{dx} \quad (3)$$

where k is the hydraulic conductivity of GCLs (m/s).

By simultaneous solution of Eqs. 2 and 3, the hydraulic conductivity (m/s) of two montmorillonite parallel layers at the exchangeable-cation i, k_{i} can be expressed as:

$$k_i = \frac{r_{\mathrm{p}}}{12\mu_{\mathrm{wi}}}(2s_i)^2 = \frac{r_{\mathrm{p}}}{3\mu_{\mathrm{wi}}}{s_i}^2 \quad (4)$$

Moreover, for a given bentonite, μ_{wi} and r_{p} are constant. Therefore, k_{i} depends only on s_{i}.

Komine & Ogata (2003, 2004) proposed the parameter "swelling volumetric strain of montmorillonite $\varepsilon_{\mathrm{sv}}$." The $\varepsilon_{\mathrm{sv}}$ is the percentage volume increase of swelling deformation of montmorillonite. This parameter is defined by:

$$\varepsilon_{\mathrm{sv}} = \frac{V_{\mathrm{V}} + V_{\mathrm{sw}}}{V_{\mathrm{m}}} \times 100(\%) \quad (5)$$

where V_{m} is the volume of montmorillonite in the buffer material, V_{v} is the volume of voids in the buffer material, and V_{sw} is the maximum swelling deformation of the buffer material at constant vertical pressure ($V_{\mathrm{sw}} \geq 0$).

Meanwhile, from the viewpoint of the behavior of montmorillonite, $\varepsilon_{\mathrm{sv}}$ can also be expressed as (Komine & Ogata, 2003, 2004):

$$\varepsilon_{\mathrm{sv}} = \frac{H_1 - H_0}{H_0} = \frac{d - R_{\mathrm{ion}}}{t + R_{\mathrm{ion}}} \times 100\% \quad (6)$$

where t is the thickness of the montmorillonite layer (m) and R_{ion} is the ionic radius.

In addition, an equation that accommodates the influences of the exchangeable-cation composition of bentonite by parameters giving the numbers of Na^+, K^+, Ca^{2+}, Mg^{2+}, and the radius of exchangeable cations after montmorillonite swelling can be obtained as:

$$s_i = \varepsilon_{\mathrm{sv}}[t + (R_{\mathrm{ion}})_i] + (R_{\mathrm{ion}})_i \quad (7)$$

where $(R_{\mathrm{ion}})_i$ is the ionic radius of the exchangeable-cation i.

Evidence of self-similar microstructures were found in bentonite (Mandelbrot, 1982; Pusch & Yong, 2003; Pusch, 1999). Xu et al. (2014a, b) analyzed the deformation and fractal behavior of bentonite by relating the elastic and fracture behaviors as proposed in the following model:

$$\varepsilon_{sv} = \frac{V_w}{V_m} = K\sigma_c^{D_s-3} \tag{8}$$

where V_w is the water volume adsorbed by montmorillonite and V_m is the volume of montmorillonite (Xu et al., 2004). K is the montmorillonite expansion coefficient, σ_c is swelling stress, and D_s is the surface fractal dimension of montmorillonite in bentonite.

The montmorillonite fraction in bentonite can be measured using XRD. Meanwhile, in the GCL permeation tests, the bentonite is completely saturated at a constant confining pressure (ASTM D6766). This indicates that a relationship exists between the structure of a montmorillonite mineral after swelling (d, t, and R_{ion}, etc.) and a confining pressure (σ_c). Therefore, from the combined Eqs. 4, 7, and 8, the hydraulic conductivity of GCLs can be expressed by the structure of a montmorillonite mineral after swelling and the confining pressure as follows:

$$k_i = \frac{r_p}{3\mu_{wi}} \times \{K \times \sigma_c^{D_s-3} \times [t + (R_{ion})_i] + (R_{ion})_i\}^2 \tag{9}$$

Among these, K can be estimated as follows (Xu et al., 2003):

$$K = \frac{(D_s - 2)CF(2\tau\cos\alpha)^{2-D_s}}{V_m} \tag{10}$$

where C is the Hausdorff measurement of the fractal surface (Pfeifer & Schmidt, 1988), F is the free energy of the chemical solution, τ is the surface tension, and α is the contact angle of the adsorbed liquid with the material.

Some researchers have introduced 0.1 mol/L as the threshold to distinguish the stronger from the weaker solutions (Vasko et al., 2001; Jo et al., 2005). Xu et al. (2014a, b) found that K varies with the concentration of the permeating solution due to the limitations of the bilayer model. The present theoretical models, therefore, will be represented by dividing the chemical salt solution concentration into two zones:

$$k_{pre} = \begin{cases} k_a, C = 0 - 0.1 \text{ mol/L} \\ k_b, C > 0.1 \text{ mol/L} \end{cases} \tag{11}$$

where k_{pre} (m/s) is the hydraulic conductivity of GCL in the theoretical models; k_a (m/s) is the hydraulic conductivity of GCL when the solution concentration is 0–0.1 mol/L; k_b (m/s) is the hydraulic conductivity of GCL when the solution concentration is > 0.1 mol/L.

Based on the theory of Frenkel-Halsey-Hill (FHH), D_s can be given by (Avnir & Jaroniec, 1989; Neimark, 1990; Yin, 1991):

$$D_s = 3 + \frac{\ln V_{ads} - B}{\ln(\ln\frac{P_o}{P})} \tag{12}$$

where V_{ads} is the gas volume adsorbed at equilibrium pressure (cm^2/g), B is an FHH constant, P_0 is the saturation pressure of the adsorbate, and P is the nitrogen partial pressure.

Rao and Thyajaraj (2007) investigated the effect of the inflow of sodium chloride solutions on the swell compression behavior of compacted expansive clays under a range of external loads. The ratio of total vertical stress to swell stress determined the nature of strains experienced by the compacted clay specimens subjected to direct inundation with salt solutions. Therefore,

$$\sigma_c = \sigma_v + \sigma_\Pi \tag{13}$$

where σ_c is the total net vertical stress (Rao & Thyajaraj, 2007), σ_v is vertical confining stress, and σ_Π is osmotic stress.

Using partition theory, Xu et al. (2014a, b) proposed that σ_Π can be written as

$$\sigma_\Pi = \Pi(\frac{\sigma_v}{\Pi})^{D_{s-2}} \tag{14}$$

where Π is osmotic suction and can be calculated through the Van't Hoff equation as (Colin et al., 1985; Lang, 1967)

$$\Pi = Ac\xi RT \tag{15}$$

where A is the Van't Hoff coefficient, c the concentration of salt solutions, ξ the cation valence, R the universal gas constant, and T the absolute temperature.

Substituting Eqs. 13 and 14 into 9 yields:

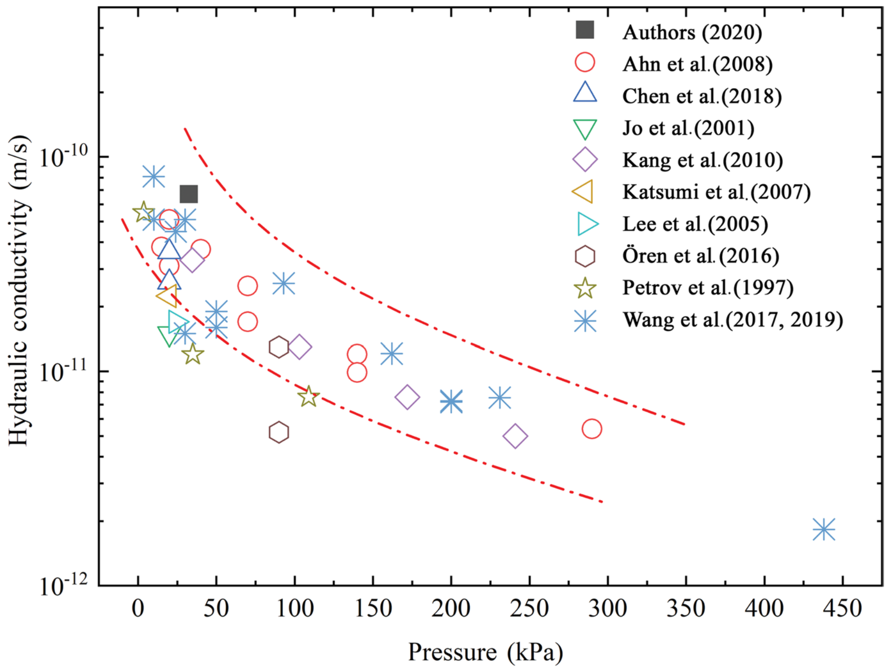


Fig. 2 The experimental hydraulic conductivity of GCLs under different pressures

$$k_i = \frac{r_p}{3\mu_{wi}} \times \{K \times [\sigma_v + \Pi(\frac{\sigma_v}{\Pi})^{D_s-2}]^{D_s-3} \times [t + (R_{\text{ion}})_i] + (R_{\text{ion}})_i\}^2 \tag{16}$$

From Eq. 9, when the confining pressure (p) increases, the hydraulic conductivity decreases. This is consistent with many experimental studies (Benson et al., 2018; Kang & Shackelford, 2011; Malusis et al., 2015; Shackelford et al., 2010; Zhu et al., 2015) as shown in Fig. 2. Shackelford et al. (2010) explained that the average confining pressure leads to a reduction in the void ratio, which results in a reduction of the hydraulic conductivity. In addition, from Eqs. 15 and 16, the increase in solution concentration will lead to an increase in Π and then, accordingly, an increase in the hydraulic conductivity. This is consistent with the experimental study of Jadda and Bag (2020), who found, using SEM, that the solution leads to the limitation of the bentonite swelling and results in the large pore size (comparing Fig. 3a through c in the red boxes), which increases the hydraulic conductivity.

Considering the main exchangeable cations (Na^+, Ca^{2+}, K^+, and Mg^{2+}) in bentonite, the hydraulic conductivity of bentonite, k_{pb}, can be approximated by

$$k_{\text{pb}} = \frac{1}{CEC} \sum_{\substack{i = Na^+, Ca^{2+} \\ K^+, Mg^{2+}}} [EXC_i k_i] \tag{17}$$

where CEC is the bentonite cation exchange capacity (meq/g) and EXC_i is the CEC (meq/g) with respect to the i^{th} ion

Introducing the montmorillonite fraction in bentonite, C_{m}, the hydraulic conductivity of GCLs, k_{pre}, can be written as:

$$k_{\text{pre}} = C_m \times k_{pb} \tag{18}$$

Considerable research has shown that the hydraulic conductivity of GCLs increased in the stronger solutions (Petrov & Rowe, 1997; Jo et al., 2001; Lee et al., 2005; Scalia & Benson, 2011; Xu et al., 2014a, b). Other studies (Mesri & Olson, 1970; Yong & Mohamed, 1992; Egloffstein, 2001) reported a positive ratio between the hydraulic conductivity of GCLs and the square root of cationic concentration. Therefore, the hydraulic conductivity at 0.1 mol/L is being used to represent the hydraulic conductivity at concentrations > 0.1 mol/L. The following equation for predicting

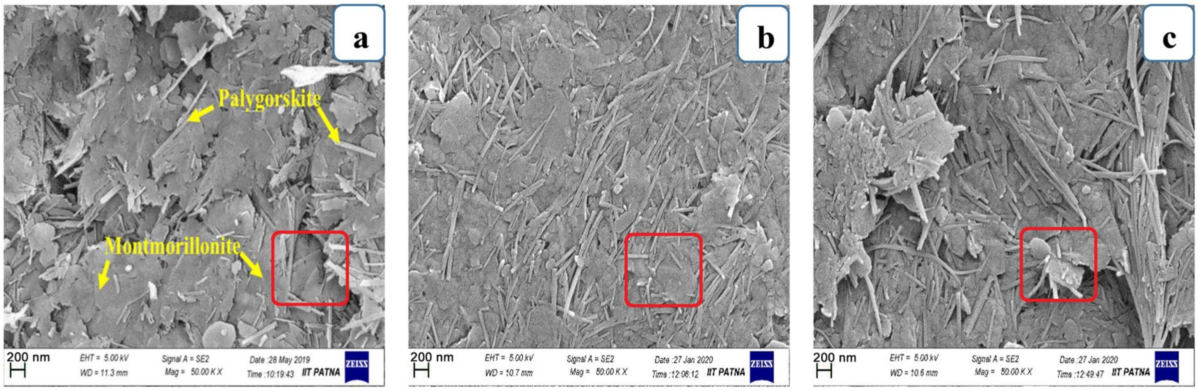


Fig. 3 Scanning Electron Microscope images of the bentonites: (a) Divalent bentonite powder sample, (b) Divalent bentonite in DI water, **(c)** Divalent bentonite in 1.0 mol/L NaCl (Jadda and Bag, 2011)

the hydraulic conductivity of GCL in the stronger solutions (> 0.1 mol/L, Bouazza et al., 2006) can be derived:

$$k_{pre} = \sqrt{\frac{c}{0.1}} \times (k_{pre})_{0.1} \quad (19)$$

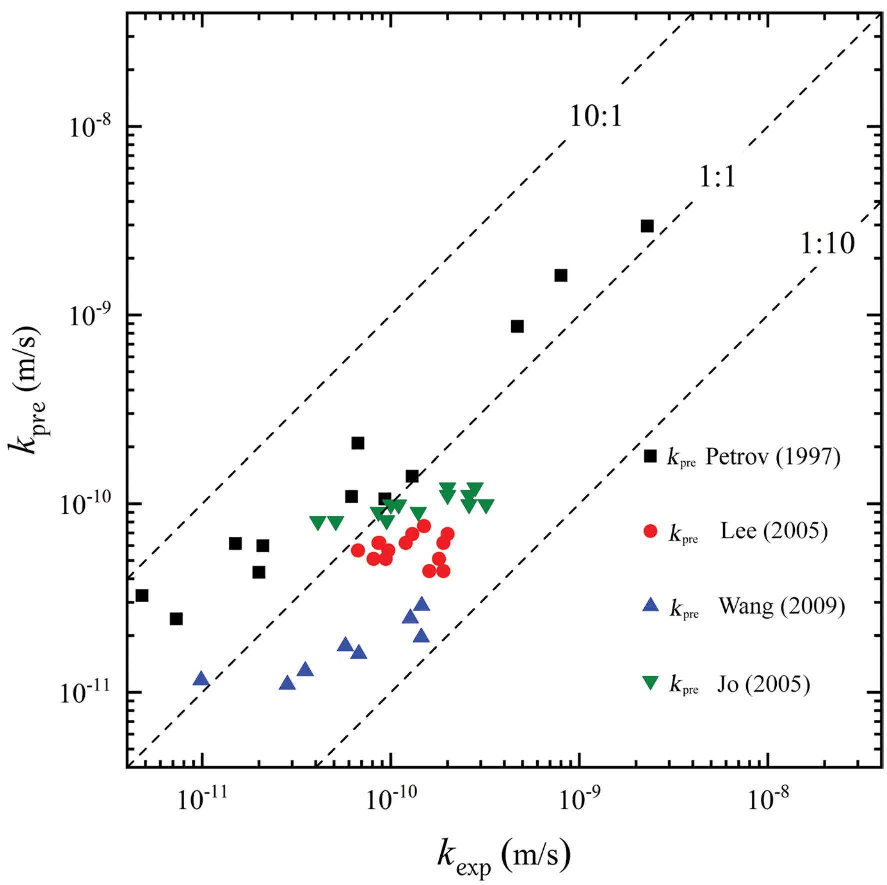


Fig. 4 Hydraulic conductivities of GCL: predicted model (k_{pre}) versus experimental data (k_{exp})

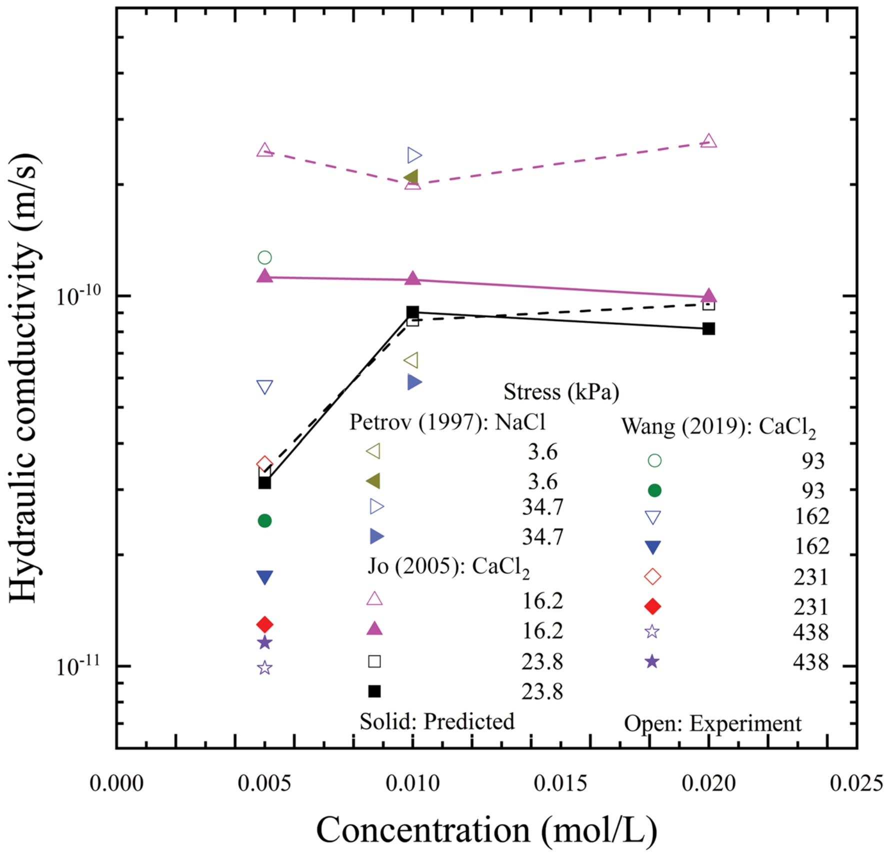


Fig. 5 Hydraulic conductivities in weak and medium solutions

Verification of the Analytical Model

To validate this theoretical model of GCL permeability coefficients, the theoretical values obtained from Eqs. 18 and 19 were verified by the experimental results of Petrov and Rowe (1997), Jo et al. (2005), Lee et al. (2005), and Wang et al. (2019a, b, c). All the parameters used in the formula are the same as those in the experiments. An example calculation is presented in Appendix I. The comparisons for different cases are summarized in Figs. 4, 5 and 6. In Fig. 4, the experimental data are represented by the x-axis and the predict data are represented by the y-axis. This ’One to one’ graph is common to illustrate the deviation of the hydraulic conductivities of GCL (Benson et al., 2018; Chen et al., 2019; Li et al., 2020). The dashed line indicates the specific deviation of hydraulic conductivities of GCL (e.g. 1:1, 2:1, 1:2). The closer to the 1:1 dash line, the smaller is the deviation of the hydraulic conductivity of GCLs. The deviation of the hydraulic conductivity of GCLs in the logarithmic coordinate system is generally between 1:10 and 10:1 (Shackelford et al., 2010; Wang et al., 2019a, b, c). It is one order of magnitude deviation, which is a conventional deviation in illustrating the hydraulic conductivity of GCL.

An Empirical Model of GCL Hydraulic Conductivity

Kolstad et al. (2004, 2006) proposed a relatively simple empirical model that can be used to estimate the hydraulic conductivity of GCLs based on experimental data. It is a function of ionic strength and relative abundance of monovalent and divalent cations (*RMD*). An adjustment model relating these parameters was developed through stepwise regression analysis (Draper & Smith, 1981) using a significance level of 0.05:

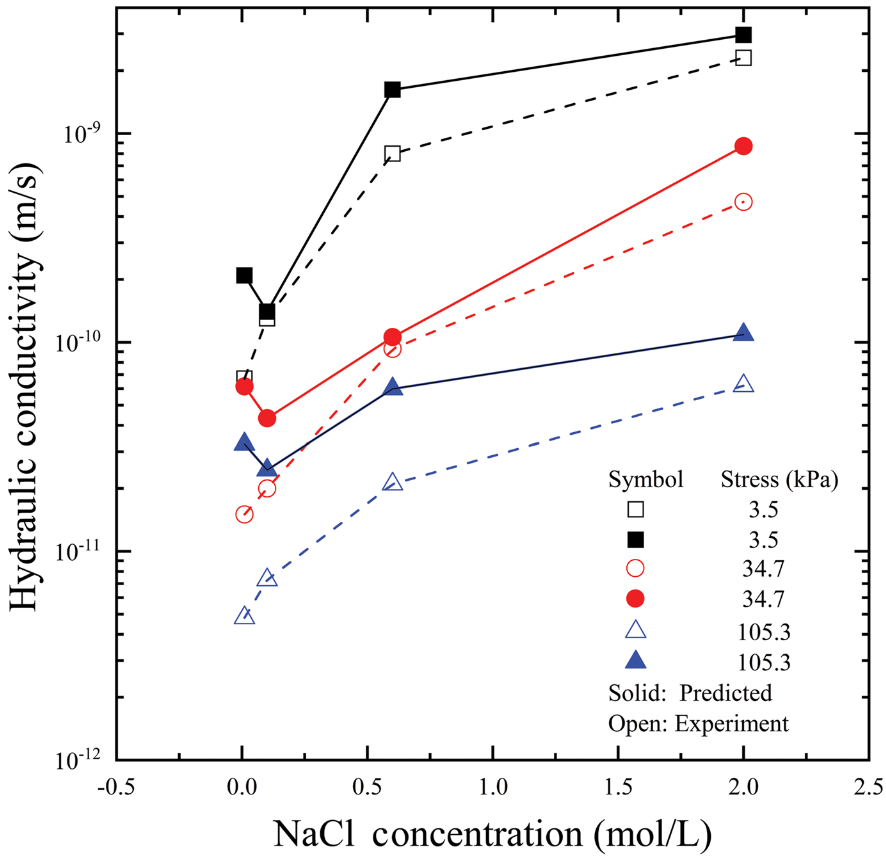


Fig. 6 Hydraulic conductivities at different pressures (NaCl)

$$\frac{\log K_{exp}}{\log K_{DI}} = 0.965 - 0.976I + 0.0797RMD + 0.251I^2RMD \tag{20}$$

where K_{exp} is the hydraulic conductivity to the inorganic chemical (m/s); K_{DI} is the hydraulic conductivity of deionized water (m/s); and I is the ionic strength. The influence of confining pressure cannot be considered in this model. To address the coupling of mechanical and chemical processes, the model accounted for the influence of the confining pressure by using regression techniques on extensive experimental data (Bradshaw et al., 2016; Chen et al., 2019; Jo et al., 2005; Lee & Shackelford, 2005; Petrov & Rowe, 1997); a good relationship between hydraulic conductivity of deionized water and the total net confining stress can be introduced conveniently as:

$$\log K_{DI} = -15.54 + 5.52\log(10.20 - 1.89\log\sigma_c)(r^2 = 0.99) \tag{21}$$

Substituting Eq. 21 into 20 yields an empirical hydraulic conductivity which combines mechanical and chemical processes:

$$\begin{aligned}\log K_{emp} &= (0.965 - 0.976I + 0.0797RMD + 0.251I^2RMD) \\ &\times(-15.54 + 5.52\log(10.20 - 1.89\log\sigma_c))\end{aligned} \tag{22}$$

Using the experimental data from Jo et al. (2005), details from example calculations of k_{pre} and k_{emp} are shown in Appendices I and II, respectively.

The deviation of the two models can be compared by the standard deviation of the sample

$$s = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2} \tag{23}$$

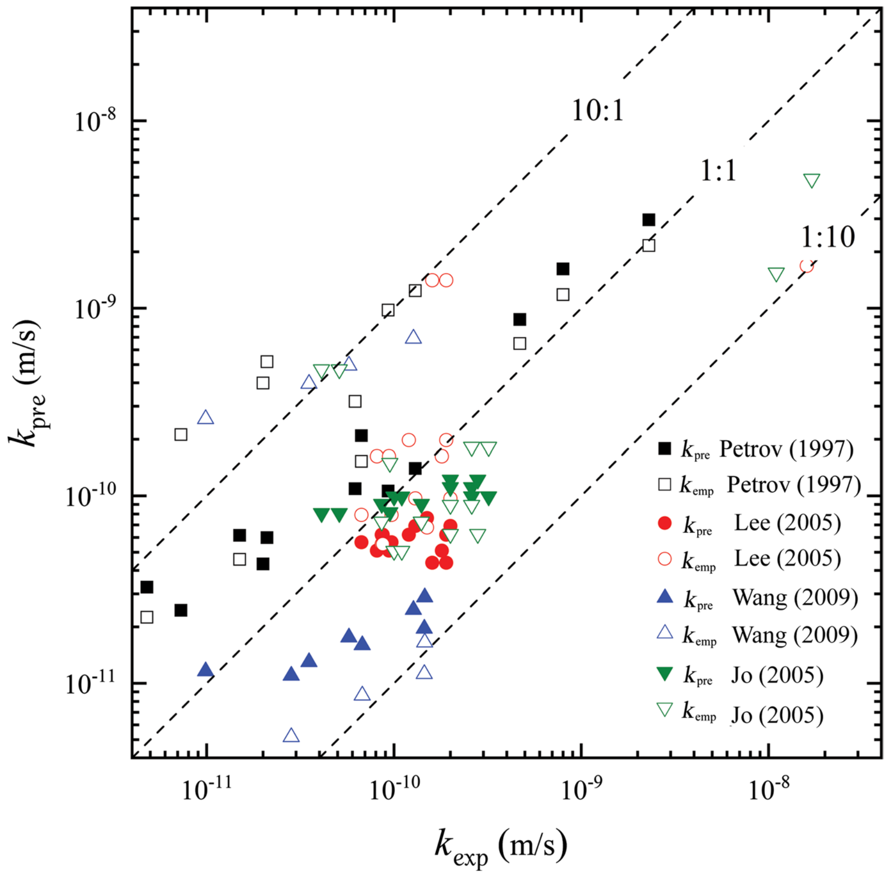


Fig. 7 Hydraulic conductivities of GCL: predicted model (k_{pre}) versus empirical model (k_{exp})

where N is the number of samples, x_i is the i^{th} sample, and $\bar{x}$ is the sample average.

Comparison of the deterministic and the empirical models is summarized in Fig. 7 and in Table 1, using the standard deviations of the two models. All other data used in Fig. 7. were calculated using the method shown in Appendices I and II, and summarized in Appendix III. From Fig. 7 and Table 1, the predicted model is more accurate while the empirical model is simpler.

Table 1 The sample standard deviation in two models

Model	N	s
Predicted Theoretical Model (k_{pre})	102	3.9
Empirical Model (k_{emp})	121	8.3

Conclusions

A simplified mathematical expression was developed to predict the hydraulic conductivity of GCLs. Two external factors were identified: confining pressure and concentration of permeating salt solutions. Six internal factors were identified: exchangeable cations; ionic radius in bentonite; surface fractal dimension of montmorillonite; the distance between two montmorillonite layers (m), exchanged with cation i (i denotes the primary exchangeable cations, such as Na^+, Ca^{2+}, K^+, and Mg^{2+} in bentonite), after swelling; and the density and coefficient of viscosity of the interlayer fluid, which is often controlled or measured during construction of GCLs liners.

The proposed prediction model (k_{pre}) has a maximum deviation of ~1:10–10:1 compared to experimental results (k_{exp}), and the empirical model (k_{emp})

has a mean deviation of ~1:15–15:1 compared to experimental results (k_{exp}). The theoretical model is more accurate than the empirical model, especially in the case of high effective stress. The writers caution, however, that the practical regression empirical model should not be used as a substitute for hydraulic conductivity assessment in the field and laboratory.

This study presents a theoretical model that predicts the hydraulic conductivity of GCL. Although the predictions have been shown to agree well with experimental data, one must remember, however, that with theoretical models, taking into account all of the actual conditions in the calculation process is complex. The theoretical model was developed based on the DDL and fractal theory, and the assumptions inherent in those theories will affect the analytical model accordingly. Therefore, more details and suitable hypotheses should be considered when this theoretical model is used in the field and in practice.

Note also that this study was developed assuming room temperature conditions only. However, some studies have shown that the temperature will affect the expansion of clay minerals (Altoé et al., 2016; Gates et al., 2017; Michels et al., 2020), which is the term s_i in Eq. 4, and the predicted hydraulic conductivity accordingly. Therefore, the effect of temperature on the permeability coefficient should be addressed in future work.

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Declarations

Conflict of Interest The authors declare that they have no conflict of interest.

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