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TECHNICAL ARTICLE

Tailings Dam Breach Analysis: A Review of Methods, Practices, and Uncertainties

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Abstract
Recent catastrophic failures of tailings storage facilities have highlighted the critical roles that dam engineers can play in ensuring public safety, and have motivated the mine waste industry to assess and improve the practice of tailings dam breach analysis (TDBAs). As industry moves towards a standard of no catastrophic failures, it is critical that practitioners, owners, and operators have a unified understanding when conducting TDBA, in order to provide a high level of confidence within communities and environments surrounding operational or closed facilities. Currently, uncertainties exist surrounding the industry's standard practice in conducting appropriate TDBA. This paper provides a summary of the currently available approaches and models for TDBA and when it is appropriate to use a particular method. A critical review of key challenges of TDBA (release volume estimate, hydrograph development, and routing the breach hydrograph downstream) is also provided. This paper aims to be a thorough summary of what is known about TDBA and a reference source for engineers and researchers.

Keywords TDBA · FLO-2D · Hydrograph · Dam break · Dam failure · Release volume

Introduction

Public concern over the safety of tailings storage facilities (TSFs) has been growing as recent tailings dam failures have occurred in Canada (2014), Mexico (2014), and Brazil (2015 and 2019), resulting in significant impacts to people's livelihoods, lives, and the environment, as well as significant costs to the dam owners. Breaches of tailings dams can produce sudden releases of water and sediment, and can take the form of an outburst flood, mud flow, or a combination of both, depending on the sediment concentration of the tailings flow.

Hundreds of dam breaches have been recorded worldwide during the last century (Shahid and Qiren 2010) and their consequences have been significant and, in many cases, catastrophic. Consequences of historic events have included loss of life, changes in downstream fluvial geomorphology and slope stability, and widespread contamination, resulting in loss of terrestrial and aquatic habitat. Despite high direct and indirect costs associated with these events, limited industry and scientific efforts have been focused on the study of their behavior and implications for mine hazard assessments and risk management.

Dam breach inundation studies are produced for many TSFs as part of the life-of-mine reporting requirements. These studies prepare inundation maps that are used to estimate the potential consequences of tailings dam failure and to develop emergency response and preparedness plans. This work is typically carried out using national guidelines, such as the Canadian Dam Association Dam Safety Guidelines in Canada (CDA 2013) and international dam safety guidelines (e.g. the International Commission on Large Dams—ICOLD). However, the breach analysis methods that are commonly applied in these studies

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were developed for clear water flows and do not consider the complexity of tailings dam failures. Available runout (distance that tailings fluid travels downstream) analysis methods have not been properly calibrated or validated to simulate flowing tailings. Consequently, these methods do not properly characterize the magnitude and intensity of possible tailings dam breach and runout scenarios.

A new draft guideline specific to tailings dam breach analysis (TDBA) has been recently prepared by the CDA (2020). In addition, various mine owners have developed internal guidelines for TDBAs. These new guidelines will likely compel mine owners and their consultants to address some of the unique characteristics of TSF breaches. To improve existing methods for assessing hazard and risk, a better understanding of tailings dam breach mechanisms and runout characteristics is required. Consultants, mine owners, and academics are acknowledging the shortcomings of the current assessment processes and are working to make improvements in this field. Research will improve the quality and accuracy of TDBA assessments to better guide TSF design and the necessary emergency response plans.

Common models used for TDBA do not have the capabilities to model the stability, liquefaction, or erosion of the tailings, so assumptions are made for the quantity of tailings released during such failures. While models have been developed to aid in estimating breach parameters, most have not been developed for tailings dams and the mechanics that may be involved in the stability or erosion of tailings during a dam breach.

This study provides an overview of current best practice methods and an insight into the missing important factors, and reviews the available literature for TDBA. Our intent was to fill in gaps in the current industry standard practice of TDBA and to critically review methods for breach hydrograph development as well as downstream flood routing. Following this introduction, a general overview of the current practice in TDBAs is presented, including failure mode analysis and routing the breach flood downstream. The section after that identifies uncertainties in TDBA that must be considered while considering the limitations of such studies. A summary of the available methods to estimate the tailings release volume from a TSF is also reviewed, along with their strengths and limitations. Modeling exercises used to obtain the tailings dam breach release hydrograph and to route the hydrograph downstream of the breach location are then discussed. These two sections also provide a concise critical review of the common applications that are used in industry for TDBA. Some other key topics and importance of research and development in TDBA are also presented with a few examples of state-of-the-art research programs on the topics discussed in this paper.

Review

Available Guidance

Many guidelines recognize the need to include a TDBA as part of the dam design of a TSF (e.g. ANCOLD 2012; CDA 2013). In most cases, these guidelines do not include methods, criteria, or procedures for developing a compliant TDBA; rather, they provide basic overviews on the components that should be included, how detailed these are required to be for particular phases of design or emergency response planning, and how to utilize the results of the corresponding DBA. Recently, the CDA released a bulletin (CDA 2020), which provides relevant steps to follow when undertaking a TDBA and references to relevant studies. This technical bulletin provides a good overview on best practice steps to follow, and will likely represent a baseline for commencing a TDBA. Unfortunately, no specific local or global guidance is available or commonly accepted, at this time.

Steps in Undertaking a TDBA

The general steps for undertaking a TDBA for each project are similar to those defined in guidelines such as the CDA (2020) bulletin:

1) **Define the objective and scope:** this step defines the objective of the TDBA, that is, how the result will be used (e.g. consequence classification or emergency planning). This will assist in assessing the appropriate level of detail required for the assessment.
2) **Conduct background information assessment and review**: this step includes investigating the design, operational, and site conditions (including nearby populations or sensitive environmental receptors), characterization of tailings, field and laboratory investigations, and analysis of flow liquefaction susceptibility, topographic data, and hydrologic data.
3) **Assess failure modes and dam failure scenarios:** this step involves engineering judgment to assess potential failure modes under both sunny-day (normal) operating conditions and rainy-day (flood) conditions under all relevant phases of the TSF development (construction, operation, closure, and post-closure). The bulletin suggests that for a consequence classification, all potential failure modes should be considered, regardless of their likelihood, whereas some failure modes may be classified as non-credible for emergency response planning.
4) **TDBA assessment cases**: the bulletin provides four potential cases when conducting TDBAs. The four

cases are dependent on whether there is a supernatant pond and whether the tailings are potentially liquefiable (Table 1). Once the tailings have been classed as either Case 1A, 1B, 2A or 2B (CDA 2020), engineering assessments can be undertaken accordingly. In some cases, no hydrologic analysis, breach analysis, or runout analysis is required.

5) **Engineering analyses**: after deciding the assessment case, the following engineering analyses should be undertaken; hydrologic, breach, runout, deposition, slope failure, and sensitivity analysis, along with inundation and deposition mapping. Not all studies are required for all cases.
6) **Documentation and reporting**: this is a critical step in ensuring that all assumptions, limitations, and results of a TDBA are appropriately recorded for future use and for clear understanding for any future studies and analyses.

Failure Mode Analysis

There are different failure modes for embankment dams, such as hydraulic failure (e.g. overtopping and top erosion), seepage failure (e.g. piping) and structural failure (e.g. foundation and slope failures). To undertake any TDBA, each of the failure modes needs to be evaluated. Generally, only one failure mode will be followed through for the detailed modeling phase of a TDBA. The two most typical failure modes considered in modeling packages are *piping failure* and *overtopping* (CDA 2013). Piping failure can occur at any depth of the dam and can be triggered by various mechanisms such as poorly compacted materials in the dam, use of unsuitable construction materials, poor drainage, or pipes poorly installed through dams, resulting in preferential pathways and unintentional fluid flow. Overtopping can be caused by an extreme storm event that is larger than the design capacity of the dam (underestimated probable maximum flood), foundation settlement (resulting in smaller freeboards than intended), or mismanagement of the facility, resulting in smaller than designed freeboards. A failure mode that is not commonly included in breach parameter estimation software is liquefaction/earthquake failure. This may occur due to a loss of strength of tailings, and is often triggered by either static or dynamic loading. The failure mechanism is used for the development of the breach hydrograph (release speed). When a sudden failure occurs, as with failures associated with liquefaction/earthquakes, the breach hydrograph should reflect the release volume being rapidly discharged from the breach location. This is an area requiring additional research to better understand the shape of release hydrographs under these conditions and should be incorporated as a failure mechanism in software. TSFs that are raised upstream are most susceptible to failure after a loss in tailings strength, as the stability of the structure depends on this strength. Once the stored material loses strength, the TSF can fail catastrophically. A potential reason that software and corresponding regressions do not incorporate this failure mechanism could be that of all of the water storage facilities, only TSFs are raised upstream. But, with so many potential modes of failure, it is critical that designers investigate all triggering mechanisms in sequences that could lead to one of the failure modes.

Failure Modes and Effects Analysis and Failure Modes, Effects, and Criticality Analysis

Various guidelines and procedures have described how failure mode analyses should be structured. The Australian/New Zealand Standard on risk analysis of technological systems (AS/NZS 1998) and the Australian National Committee on Large Dams (ANCOLD 2003) suggest that failure modes and effects analysis (FMEA) and failure modes, effects and

Table 1 Tailings dam breach assessment cases (Source: CDA 2020)

Presence of supernatant pond near the dam	Potential for tailings runout as a result of flow liquefaction[a]	
	Yes	No
Yes	Case 1A: Liquefied tailings with a pond: Dam breach with flow of fluids and eroded and liquefied flowable tailings contributing additional volume of materials released	Case 1B: Non-liquefied tailings with a pond: Dam breach with eroded tailings, transported and deposited by the flow of fluids
No	Case 2A: Liquefied tailings without a pond: Dam breach resulting from slope failure with mudflow or debris type flow of liquefied flowable tailings (depending on the degree of saturation)	Case 2B[b]: Non-liquefied tailings without a pond: Slope failure of the dam

[a]Flow liquefaction of tailings could be induced by any potential trigger (static or cyclic/seismic) including shear strains in the tailings as a result of the dam breach (e.g., lateral unloading)

[b]Hydrotechnical analyses or inundation mapping similar to other three cases would not be required for Case 2B. Landslide runout analysis may be more appropriate

criticality analysis (FMEACA) should be used as a *screening* process to assess whether there is need to carry out more rigorous analyses (Chapman and Williams 2019).

An FMEA is described as *a qualitative technique by which the effects of individual component failures are systematically identified*. It uses tools such as fault and event trees to assess a potential failure and to develop an exhaustive identification of the components and their failure modes (ANCOLD 2003). FMEAs depend on subjective identification and assessment of potential failure mechanisms.

An FMEACA is an extended version of an FMEA that incorporates the additional step of determining an index rating to assess the criticality of each event identified in the FMEA. The ANCOLD (2003) Guideline on Risk Assessment provides detailed processes and example templates for conducting both FMEAs and FMEACAs.

Quantitative Risk Assessment

A quantitative risk assessment (QRA) systematically combines all potential faults that could result in a TSF failure and evaluates the possible consequences of such a failure (Chapman and Williams 2019). This quantitative failure mode assessment technique is more rigorous than the FMEA and FMEACA methods as it incorporates the effects of physical events on the system, and can account for human interactions. QRAs can be complex to develop and require a detailed understanding of the TSF to adequately prepare, but they provide the most comprehensive and informative method to assess TSF failure modes. An example of a QRA for catastrophic release of tailings and/or water from a (specific) TSF is presented in Fig. 1. Details on how to use various failure mode assessments and their advantages and disadvantages are summarized in Chapman and Williams (2019).

Implementation of Risk Assessments for TDBAs

It is common practice to eliminate various failure modes because they are considered non-credible. For example, in some regions of the world, earthquakes are no more likely than meteor strikes, and so can be shown to be non-credible. However, to classify a failure mode as non-credible, serious consideration should be given, and studies, in the form of an FMEA, an FMEACA, or preferably, a QRA, should be undertaken. Fault mode assessments can provide certainty that a failure mode is not only credible, but that, of all credible failure modes, it is the mode that will result in the most significant failure. If there are multiple credible failure modes and it is not clear which will have the most critical impact, designers should consider advancing multiple scenarios to preliminary modeling phases to better gauge which will have a greater influence, and hence, which will correctly classify failure consequence or provide accurate information for emergency response planning. Likewise, if all failure modes are assessed as non-credible, the failure mechanism that would have the most impact should be considered and adopted for consequence classification purposes.

It is critical that the decision by designers to proceed with a credible failure mode not be confused with the probability of such a failure occurring. More often than not, TDBAs are undertaken with a design that has been developed to counteract the potential failure modes, and hence a hypothetical worst-case failure mode is assumed and adopted for the study. When developing drawings and inundation maps, limitations and assumptions as to what the TDBA represents should be clearly indicated on the drawings and not solely within the corresponding text. With design documentation becoming more publicly accessible, it is critical that inundation maps are correctly interpreted by the public and correctly implemented by all involved, as these maps are often used without referring to the reports.

Available Commercial Software for Breach Hydrographs

An overview of software-implementing models typically used to prepare a dam breach hydrograph (and routing it downstream, in some cases) for tailings dams follows (Clemente et al. 2013). Each of the models presented has advantages and disadvantages based on the specific case and the available input information. The software often requires

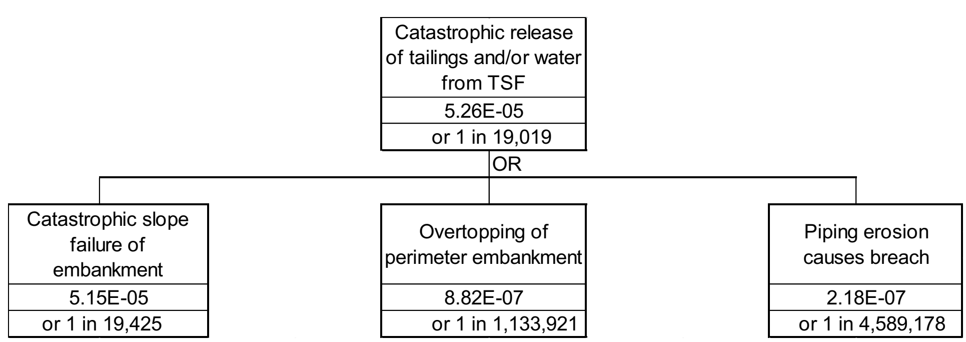


Fig. 1 Example of QRA decision tree (Source: Chapman and Williams 2019)

input data such as topography, tailings rheology (including specific gravity, viscosity, and yield stress), TSF geometry, stage-storage curves for the TSF and supernatant pond, and baseline hydrological data (CDA 2020). The objective of this overview is to summarize the capabilities, complexities, and limitations of the available models for a given TDBA. More information about these software packages can be obtained from their providers.

DAMBRK was developed by the National Weather Service (NWS; Fread 1984) and predicts the dam break wave formation and downstream progression. The model consists of three functional parts: (1) description of the dam failure mode, including the temporal and geometric description of the break; (2) computation of the outflow hydrograph through the break; and (3) routing of the outflow hydrograph through a downstream channel. In computing the peak outflow and the outflow hydrograph from the break, the program utilizes user inputs describing the geometric and temporal patterns of the reservoir and the break. After computing the outflow hydrograph, the program uses a dynamic wave method to route the flood wave in the downstream channel or valley. While commonly used to simulate Newtonian (clear water) flows, DAMBRK can also simulate routing of non-Newtonian fluids (tailings), by specifying the rheology of the fluid, such as its unit weight, dynamic viscosity, initial shear strength, and stress rate of strain. Use of DAMBRK to model tailings dam breaks is reported by Browne (2011).

FLDWAV is a computer model developed by the NWS to model flows through a single stream or network of streams (Fread 1993, 2000). The FLDWAV program is an upgrade of DAMBRK, and was designed to analyze large flood events from more than just dam breaks by using a real-time forecasting predictive model. This model has five main capabilities: (1) to model single or multiple channel flow for straight or meandering channels; (2) to model free surface flows in subcritical, supercritical and mixed flow regimes, as well as conduit flow under pressure; (3) to model clear water fluids and mud/debris fluid flows; (4) to model off-channel flow areas that may take flow storage during high flows; and (5) to model time-dependent dam breaks or control structures along with flow over and through multiple control structures. The system is based on a one-dimensional solution to the Saint–Venant equations for unsteady flow. It also allows the user to model one-dimensional unsteady non-Newtonian fluids. Use of FLDWAV to model tailings dam breaks is reported by Kunkel (2011).

HEC-HMS was initially designed to simulate the precipitation-runoff processes of drainage basins, but is also used to obtain breach hydrographs. A Monte Carlo stochastic approach is used to quantify the sensitivity of the outflow hydrograph to the uncertainty in dam breach parameters. Many plausible parameter configurations are simulated by randomly selecting individual parameter values according to an assumed probability density function. The resulting outflow hydrograph ensemble is statistically analyzed to evaluate the range of plausible breach responses, primarily in terms of the peak (maximum) outflow. HEC-HMS models the flow through the expanding dam breach as weir flow using an equation developed for water flow that does not account for the effects of viscosity. This implies that these calculations do not consider the non-Newtonian nature of the fluid, which can cause the model to overestimate the effect of a breach discharge.

HEC-RAS (2D) allows the modeling of a dam break failure, though it does not model non-Newtonian fluids. It allows unsteady two-dimensional flow routing using all of the Saint–Venant equations and is often used to compute and display downstream effects resulting from hypothetical dam failures. Given input parameters, such as ultimate breach geometry and time to breach, HEC-RAS 2D can generate the dam breach hydrograph and then simulate the resultant flood wave and downstream consequences. HEC-RAS 2D has been used for flood damage analysis and dam safety studies. Routing simulations require the following from the break model to work in HEC-RAS 2D: location, failure mode, shape and progression, formation time, trigger condition, and weir and pipe flow coefficients. HEC-RAS 2D also provides a time growth template for breaks.

RiverFlow2D is a software developed by Hydronia LLC that has built-in functionality to develop breach progression. In RiverFlow2D, the dam is defined as an internal boundary condition and is modeled as a progressive trapezoid. Full details of the breach development equations are presented in RiverFLow2D User Manuals.

FLO-2D has built-in functionality to develop breach progression. To estimate the breach hydrograph, the user requires the breach parameters described above and a user-defined release volume. Full details of the breach development equations are presented in FLO-2D User Manuals.

Historic Failures and Failure Mechanisms

A database has been developed as part of this study to assist in better understanding historic failures. 85 dams have been included in the database, which includes historic failures that have information available in at least four of the following six categories: failure date, failure location, failure mechanism/mode, dam height, construction method and estimated release volume. A large portion of the data points have been obtained from *wise-uranium* (2019), with additional data points or information researched from various other records and reports. The most common failure mechanisms/modes were from earthquake/liquefaction (19 failures), overtopping (16 failures), and slope instability (16 failures), accounting for 60% of all failures included in the study. It is noteworthy that overtopping failure is often caused by an extreme storm event that is larger than the design capacity of the dam, while erosion failure is often caused by erosion of downstream toe of the earth slope caused by misdirected spillway outlet discharge. There was no definitive correlation between dam height and release volume when separated by failure mechanism; 50% of the highest dams also had the largest volume release (liquefaction, erosion, foundation, and undefined), and the remaining failure mechanisms had larger release volumes with smaller dam heights (Fig. 2). Although all dams of 10 m or less in height had a release volume of less than 0.02 Mm^3 (million cubic meters), this trend is not observed in dam heights above 10 m, with some dams 15 m in height releasing volumes between 1.8 and 10 Mm^3.

The dataset shows that more upstream dams failed (46) than any other construction method, and most (20) failed due to liquefaction of the material (Fig. 3), triggered by either earthquakes or static loads. Failures of downstream constructed facilities only accounted for eight of the reported failures, though construction methods for 13 of the failures were not reported. The high proportion of upstream failures does not imply that upstream constructed facilities are less safe, but is more likely due to the facility's location, lack of effective local governing bodies, old facilities that were designed using outdated methods or guidelines (especially with respect to designing against failure mechanisms and storing the probable maximum precipitation), or the facility being located in an area with a high magnitude and frequency of earthquakes. Many regulatory bodies have updated guidelines and design requirements considering historic failures. For example, Chile is in region with high seismic activity, where there are frequent earthquakes with magnitudes of greater than 7.0 Mw (moment magnitude) and the world's largest recorded earthquake. With such high seismic activity, the Chilean Government has banned upstream raised facilities in the country since ≈ 1970, whereas countries with stringent standards around TSF design but very low earthquake activity and generally low rainfall (for example, Australia) still permit the design and construction of upstream raised facilities.

Shahid and Qiren (2010) reviewed tailings dam failures over the previous 100 years, and reported that rain-induced failures increased from 25% pre-2000 to 40% post-2000, which could be attributed to climate change, especially at mine sites close to the sea. Likewise, failures due to poor management accounted for 10% and 30%, respectively, for the two-time groups. This increase could indicate the rush for natural resource exploitation that compromised engineering standards in various parts of the globe. According to Rico et al. (2007), poor management includes inappropriate dam construction procedures, improper maintenance of drainage structures, deviation(s) from design, and inadequate long-term monitoring programs.

Mining operations need to continue to construct dams to store their tailings byproducts, but these dams are continuing to fail, and the failures do not discriminate amongst the phase; closed, active, old, and new, though the importance and demand of high-quality and accurate TDBAs is increasing. Industry is adapting to keep up with best practice, comply with regulations that are being developed both

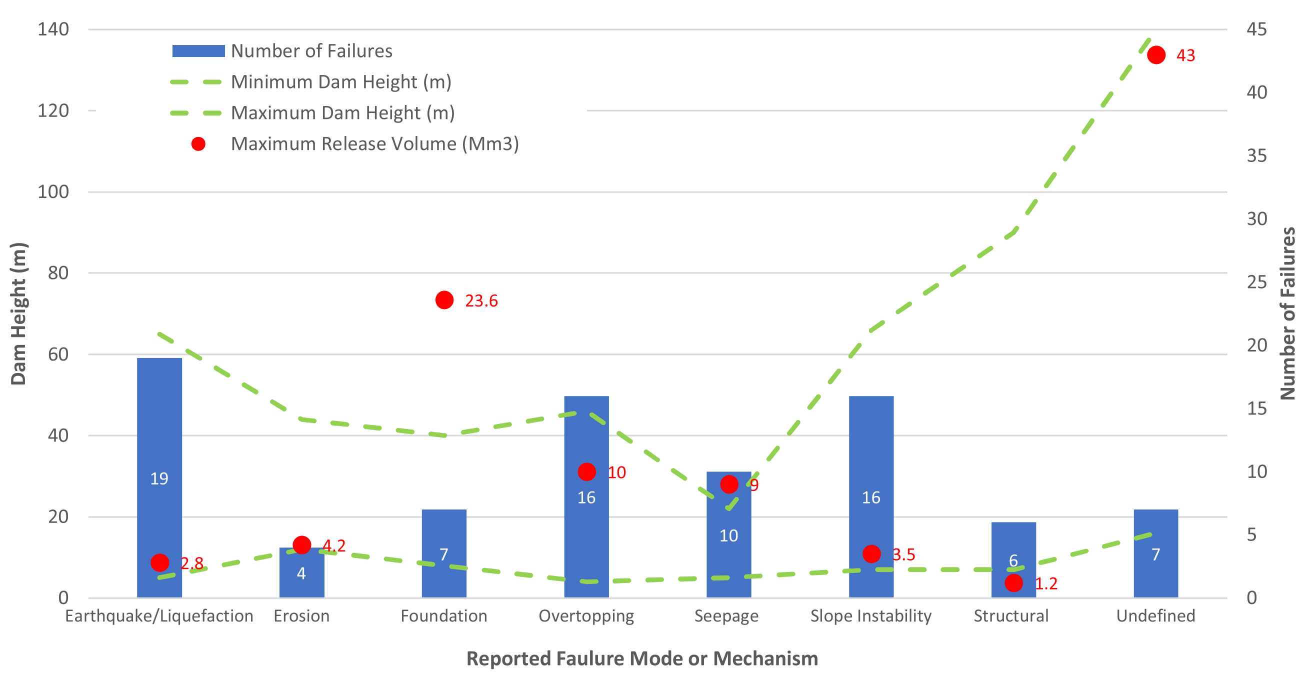


Fig. 2 Release volume in relation to dam height and failure mechanism

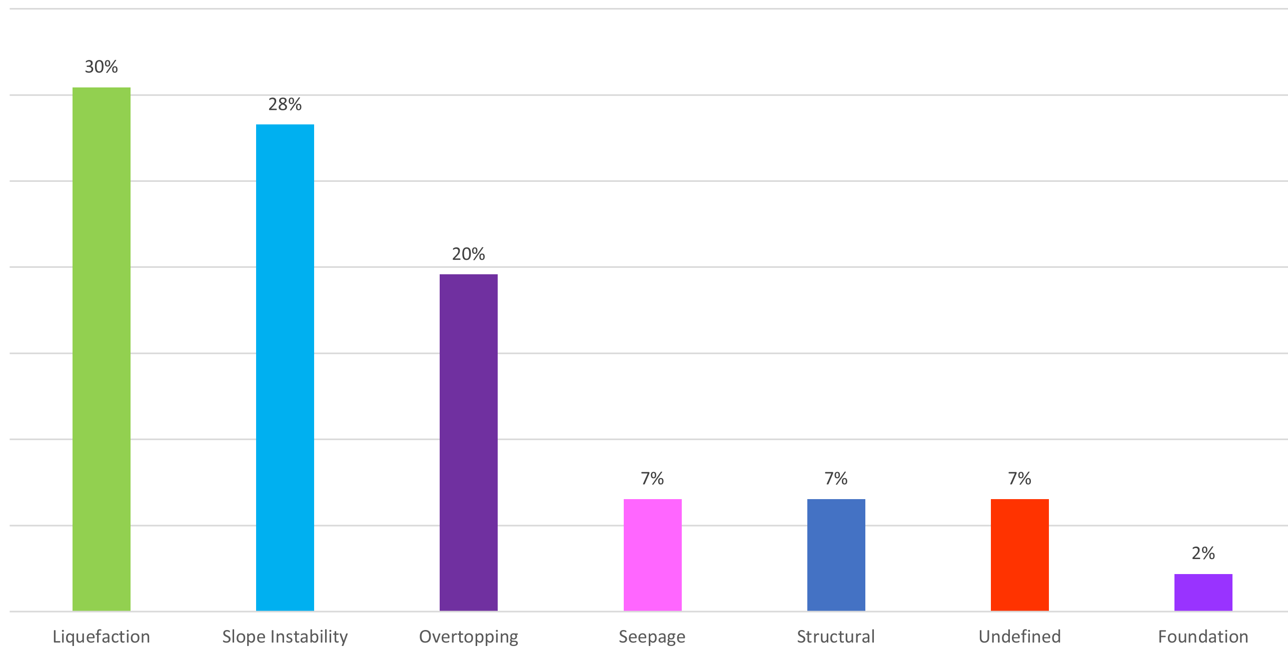


Fig. 3 Upstream dam failure mechanisms

locally and internationally, reduce the risk of populations downstream in the event of failures (Global Tailings Review 2019), and to develop and study the uncertainties that still exist in the field.

Uncertainties in TDBAs

There are several uncertainties in TDBAs that are important but were rarely considered in past studies. The following key technical challenges need to be addressed to develop an effective understanding of TDBAs in current industry practice.

Estimation of the Volume of Tailings Released During a Breach

Current practice relies mainly on two volume release methods: historic tailings dam failure shapes and slopes, and empirical equations derived from historic datasets. However, the dam characteristics, tailings rheology parameters, pond and immediate downstream topographic conditions need to be included to appropriately estimate the tailings volume that will be released during a tailings dam failure. In addition, there is uncertainty in the time that the release volume takes to discharge from the breach location, i.e. the development of the breach hydrograph (discharge vs time). A comprehensive literature review and data analysis of historic tailings dam failures is needed to develop appropriate procedures and methods for more realistic estimation of release volumes in the event of a tailings dam failure to significantly improve the tailings deposition and runout predictions downstream. This paper provides a review of available methods for estimating the release volume and the advantages and disadvantages of each method, as well as a comparison of methods using historic data.

Application of Suitable Tailings Rheology Parameters

The two key tailings rheology parameters when dealing with non-Newtonian tailings dam breach modeling analysis are yield shear stress and dynamic viscosity. Current practice relies largely on tailings design information and available field test data from various mine sites, and practitioners are required to apply professional judgment due to data variability. A stepwise technical approach to reduce the risk or uncertainty in the selection of yield shear stress and dynamic viscosity values for simulating tailings dam failures is strongly advised. This could be achieved by extensive literature review and summarization of available experimental and field data followed by sorting them based on the tailings characteristics in order to ascertain the tailings properties. In addition, as the fluid body moves downstream, accumulates sediment (eroded material, vegetation, or any infrastructure), and loses volume from deposition, material behavior and fluid rheology change. Currently, no modeling software exists that is capable of capturing this change in fluid behavior. Research into how this affects modeling results and the behavior of the mass flow is needed.

Assessing Floodway Erosion and Sedimentation

Current practice considers a fixed bed (channel and floodplain) downstream for routing dam breach flood waves. This is a simplified approach. Historic and recent tailings dam failures (e.g. the Mount Polly Dam in BC and Fundao Dam in Brazil) suggest that a tailings dam breach generates massive mud flows that cause significant channel erosion downstream of the dam due to higher shear stress and viscosity rates of their fluid compared to water. A hindcast modeling analysis based on available historic tailings dam failures is needed to quantify uncertainties and assess the possibility of implementing a mobile bed modeling approach.

Volume Release Estimation

Accurately estimating the volume of tailings released during a hypothetical tailings dam breach is inherently complicated due to the level of uncertainty associated with such an analysis. Published literature indicates that the percentage of stored tailings released ranges from 1 to 100%, with a reported average between 20 and 40% (Clemente et al. 2013). The following conceptualization is required to estimate the potential volume of tailings released during a hypothetical failure:

- The failure occurs.
- The leading edge of material leaving breach is water.
- Hydraulic erosional forces combined with the liquefaction process entrain tailings to create a tailings breach slurry.
- Once all supernatant and stored water is released, hydraulic erosional forces subside, and soil mechanics dominate.
- Liquefied tailings may continue to flow and tend to a material specific $\frac{S_u}{\sigma_v}$ (where S_u is the undrained shear strength and σ_v is the vertical stress) value, the lower bound of which is 0.05–0.1 (Reid and Fourie 2014).
- The backscarp slopes in the remaining tailings may tend to a pre-failure in-situ state, which may be near vertical.

Being able to physically model this process would lead to increased accuracy in estimating the volume of tailings released during a breach. Commercially available technology does not have the capacity to do this. There are several simplified methods for estimating the volume of tailings released: statistical regression, flowability approximation, and geometric estimation, as described below.

Statistical Regression

Several authors have published statistical analyses that relate specific tailings dam characteristics (e.g. dam height, impounded volume, percentage of tailings covered by water) to the total volume of estimated tailings released during a breach. The most recent estimate is an extension of an analysis in Rico et al. (2008) and approximates the total volume of released tailings (V_F) as 0.332 times the impounded volume (V_T) to the power of 0.95 (Larrauri and Lall 2018):

$$V_F = 0.332 \times V_T^{0.95} \tag{1}$$

The limitation of this approach is that it does not consider the geotechnical properties of the tailings and it is not applicable for impoundments with large surface areas.

Other statistical regressions are available that estimate the release volume as a function of dam height (H); for example. Chao et al. (2014), who proposed three equations for lower (Eq. 2), middle (Eq. 3), and upper (Eq. 4) bounds. Note that the lower, middle, and upper values are statistical bounds to account for uncertainties.

$$V_F(Lower) = 1052 \times H^{1.2821} \tag{2}$$

$$V_F(Middle) = 3604 \times H^{1.2821} \tag{3}$$

$$V_F(Upper) = 20419 \times H^{1.2821} \tag{4}$$

A limitation of this approach is that like the total volume regressions, it does not consider the geotechnical and geometrical properties of the tailings, and that the regressions have not been refined for varying dam heights and are often limited to dams of certain heights.

Geometric

This approach assumes a failure surface that radiates upstream from the breach location at an upward average slope (post-failure tailings slope) that is considered to be an average of the $\frac{S_u}{\sigma_v}$ value for the specific tailings material and the backscarp slopes in the remaining tailings, which tend to a pre-failure in situ state (as mentioned above). Observations from a small selection of historic tailings dam breaches indicate that this potential post-failure tailings slope ranges from 5 to 18% (≈ 4°–10°), based on Rourke and Luppnow (2015). The limitation of this method is that the estimated post-failure tailings slopes presented in Rourke and Luppnow (2015) are based on a very limited dataset and site-specific geotechnical properties are not considered.

Flowability Approximation

The concept of flowability of tailings assumes that as the solids concentration in a fluid increases, the flowability decreases. This method considers two specific tailings properties: density and specific gravity. It is important to consider that depending on the original composition of the ore body, various chemical properties can have a profound effect on the behavior of tailings. The limit at which the concentration prevents tailings from flowing in a measurable way is about 55–65% by volume. Fontaine and Martin (2015) suggest that if one assumes the water residing in the TSF instantaneously and homogenously mixes with an unknown volume of tailings during a breach, then the volume of tailings can be estimated using the above-noted range.

The limitation of this method is that it only works when the condition $\%w < \frac{1}{\%S} - 1$ is met (where %w is the percent water content and %s is the percent saturation). This will vary depending on the tailings rheology of a site, but is generally applicable for materials with in situ solids concentrations by volume that are less than 65% (the measurable flow limit). In addition, the supernatant pond volume significantly affects the release volume and so, the location of supernatant pond (in close proximity to the dam, or far away from it) needs to be assessed, and a judgment needs to be made as to whether release of pond water is feasible.

Comparison of Methods

Case Study

Each of the three methods described above (empirical by total storage volume and empirical by height, geometry, and flowability) were assessed against three historic failures for which the failure mechanism, total storage volume, dam height, and release volume were known. The three historic dams were all constructed with the upstream method, with heights ranging between 25 and 61 m, and with failure release volumes ranging between 0.6 and 3.5 Mm^3. Each case history had a different failure mechanism: liquefaction, overtopping, and slope instability. The reported dam characteristics for the selected historic failures are presented in Table 2.

The geometric method and the flowability method require additional dam characteristics and tailings properties that were not reported, so some additional parameters were assumed, based on common characteristics and engineering experience. The assumed parameters/characteristics are presented in Table 3. In addition, the pond volume for each historic failure was assumed (Table 4) as is required for the flowability method. It was observed that, with the same set of tailings characteristics but a pond volume set to a percentage of the storage volume, the percent of released material relative to the total storage volume was the same for each failure mechanism. This implies that this method relates the pond size to the volume of released material.

Each method was implemented using the dam's known characteristics and tailings parameters, and the release volumes were estimated and compared with the reported release volume. The results of the assessment are presented in Fig. 4, where the reported release volumes for each case (by failure mechanism) are also presented. Since the geometric and flowability approaches used some assumed parameters, a sensitivity assessment was carried out on these methods. The sensitivity assessments are presented below.

Sensitivity Analysis for Geometric Approach

Since the crest width and upstream and downstream slopes of the dam were assumed for this method, a sensitivity assessment was undertaken on the geometric method to consider the variability in these parameters. The crest width,

Table 3 Assumed dam characteristics and tailings parameters

Parameter	Assumed value
Upstream slope	2H:1 V
Downstream slope	3H:1 V
Crest width (m)	10
% Solids	60
Average dry density (t/m^3)	1.4
Tailings solids density (t/m^3)	2.7
Saturation (%)	100
Density of water (t/m^3)	1.0

Table 2 Case study dam characteristics

Location, year	Failure mechanism	Dam height (m)	Construction method	Release volume (Mm^3)	Total storage volume (Mm^3)
Chile, 1928	Liquefaction	61	Upstream	2.1	20
South Africa, 1994	Overtopping	31		0.6	7
Russia, 1981	Slope Instability	25		3.5	27

Table 4 Assumed pond volume

Failure mechanism	Assumed pond volume (Mm^3) (% pond of total storage)	Justification
Liquefaction (Chile)	1.0 (5)	Chile has an arid climate and high evaporation rates. It is unlikely the pond volume was higher than 5% of the total storage (20 Mm^3); enough water was present in the system to enable liquefaction of material. The reported runout distance was ~5 km, implying steep terrain and liquid material
Overtopping (South Africa)	0.7 (10)	As the failure mechanism was overtopping, it is reasonable to assume there was a large volume of water in the system. While the total storage volume was only ~7 Mm^3, overtopping generally implies high volumes of water along with mismanagement (e.g. operation error in managing inflow into the reservoir)
Slope Instability (Russia)	0.8 (3)	The total storage volume for this case study is the largest (27 Mm^3). The failure occurred in the middle of winter in Russia, where temperatures were likely freezing with snow. It would be difficult to store a large pond without freezing. The reported runout distance was ~1.3 km. The failure mechanism does not require water to be induced

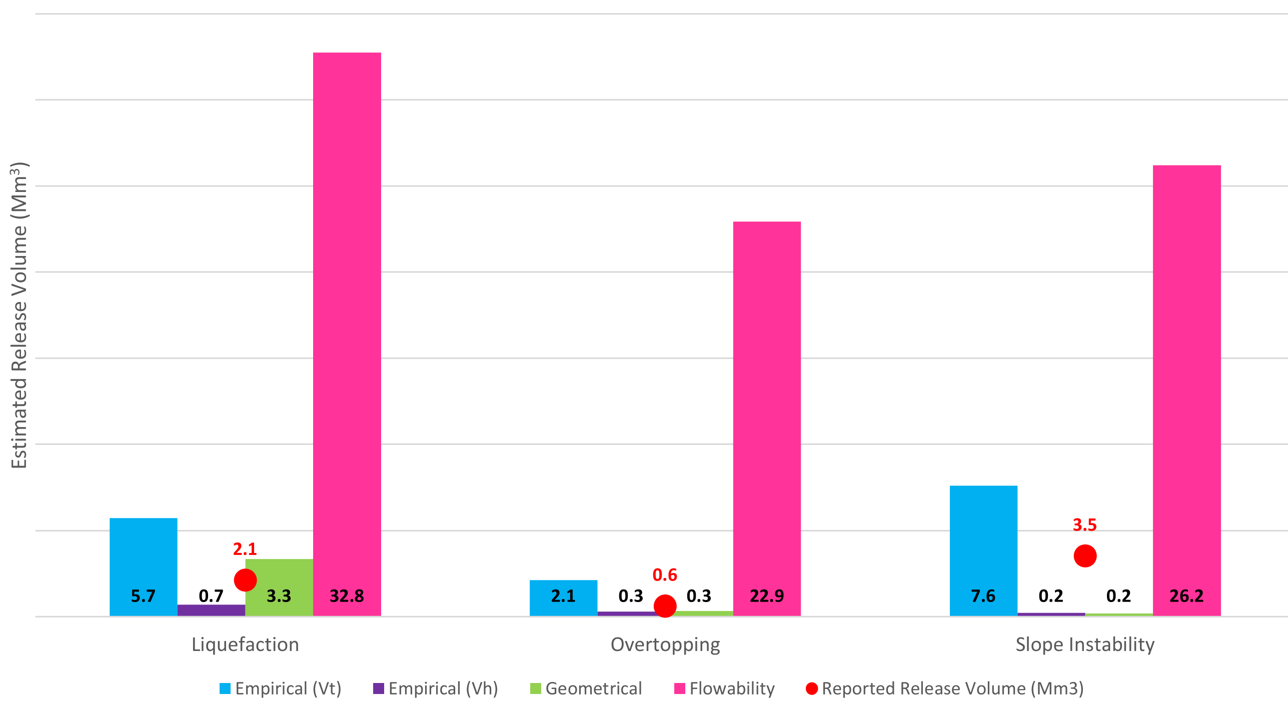


Fig. 4 Release volume by method and failure mechanism

upstream and downstream slopes, and tailings and dam breach angles (depicted in Fig. 5) were varied as part of the sensitivity assessment, with all results presented in Tables 5, 6, 7, 8 and 9 (in these tables, the baseline case refers to the results presented in Fig. 4). The crest width and upstream slope had little effect on the release volume, whereas the downstream slope, as well as the breach angle of the tailings/dam, can significantly affect it. The downstream slope of a facility is generally well understood when conducting a TDBA; however, the breach angle of the dam and tailings is less understood. It is therefore recommended, based on this sensitivity assessment, that a range of reasonable breach angles be considered and a range of release volumes within that range be considered for the TDBA. For overtopping failures, a dam breach angle of 0° is commonly adopted; to assess the sensitivity of the geometric approach, the dam breach angle for overtopping failures has been kept

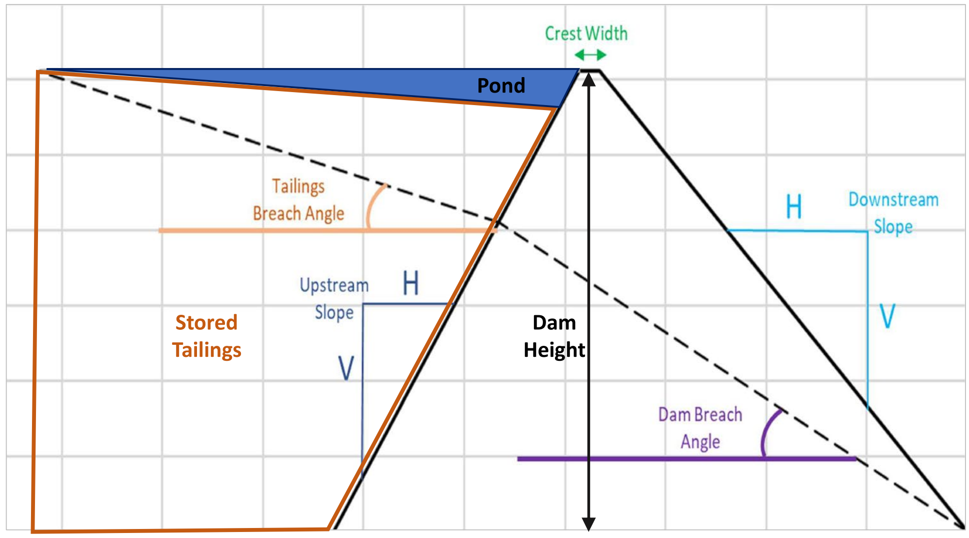


Fig. 5 Input parameters for the geometric approach

Table 5 Sensitivity analysis for geometric approach—crest width

Crest width (m)	Liquefaction (Mm^3)	Overtopping (Mm^3)	Slope instability (Mm^3)
5	2.55	0.33	0.17
10 (baseline)	2.54	0.33	0.17
20	2.52	0.32	0.17
40	2.49	0.32	0.17
Reported release volume	2.1	0.6	3.5

Table 6 Sensitivity analysis for geometric approach—upstream slope

Upstream slope	Liquefaction (Mm^3)	Overtopping (Mm^3)	Slope instability (Mm^3)
1.0H:1 V	2.56	0.33	0.17
2.0H:1 V (baseline)	2.54	0.33	0.17
3.0H:1 V	2.53	0.33	0.17
Reported release volume	2.1	0.6	3.5

Table 7 Sensitivity analysis for geometric approach—downstream slope

Downstream slope	Liquefaction (Mm^3)	Overtopping (Mm^3)	Slope instability (Mm^3)
1.5H:1 V	4.3	0.55	0.29
2.0H:1 V	3.7	0.47	0.25
2.5H:1 V	3.1	0.40	0.21
3.0H:1 V (baseline)	2.5	0.33	0.17
3.5H:1 V	2.0	0.26	0.14
Reported release volume	2.1	0.6	3.5

consistent with the dam breach angle for the liquefaction and slope instability failure mechanisms.

Table 8 Sensitivity analysis for geometric approach—tailings breach angle

Tailings breach angle (°)	Liquefaction (Mm^3)	Overtopping (Mm^3)	Slope instability (Mm^3)
4	3.8	0.47	0.24
6	3.1	0.39	0.20
8	2.7	0.35	0.18
10 (baseline)	2.5	0.33	0.17
Reported release volume	2.1	0.6	3.5

Table 9 Sensitivity analysis for geometric approach—dam breach angle

Dam breach angle (°)	Liquefaction (Mm^3)	Overtopping (Mm^3)	Slope instability (Mm^3)
4	7.7	1.1	0.5
6	5.4	0.7	0.4
8	3.8	0.5	0.2
10 (baseline)	2.5	0.3	0.2
Reported release volume	2.1	0.6	3.5

Sensitivity Analysis for the Flowability Approach

Since the flowability approach had a few assumed values, a sensitivity assessment was also done for this method. The percent solids, the average dry density, and the tailings solids density were all varied to consider the effects of the assumptions. The solids concentration was varied between 0.5 and 0.65; this was observed to have a significant effect on the release volume. With a solids concentration of 0.55, the estimated release volume was closer to the reported release volume for all failure mechanisms, but

Table 10 Sensitivity analysis for flowability approach—percent solids

Percent solids	Liquefaction (Mm^3)	Overtopping (Mm^3)	Slope instability (Mm^3)
0.5	2.2	1.5	1.8
0.55	2.8	2.0	2.2
0.6 (baseline)	4.1	2.8	3.2
0.65	8.5	5.9	6.8
Reported release volume	2.1	0.6	3.5

Table 11 Sensitivity analysis for flowability approach—average dry density

Average dry density (t/m^3)	Liquefaction (Mm^3)	Overtopping (Mm^3)	Slope instability (Mm^3)
1.3	5.0	3.5	4.0
1.4 (baseline)	4.1	2.8	3.2
1.5	3.5	2.4	2.8
1.6	3.1	2.1	2.4
Reported release volume	2.1	0.6	3.5

Table 12 Sensitivity analysis for flowability approach—tailings solids density

Tailings solids density (t/m^3)	Liquefaction (Mm^3)	Overtopping (Mm^3)	Slope instability (Mm^3)
2.5	2.3	2.6	3.0
2.6	3.9	2.7	3.1
2.7 (baseline)	4.1	2.8	3.2
2.8	4.2	2.9	3.4
Reported release volume	2.1	0.6	3.5

with a solids concentration of 0.65, the estimated release volume was drastically overestimated for each failure mechanism (Table 10). The lower the percent solids, the lower the release volume for this method.

Varying the average dry density between 1.3 and 1.6 t/m^3 affected the release volume for all failure mechanisms (Table 11). The higher the average dry density, the lower the release volume. While varying the average dry density has the potential to reduce the release volume, it does not have a significant enough effect to achieve release volumes within the range of the reported release volumes.

The tailings solids density was varied between 2.5 and 2.8 t/m^3 (Table 12), which was found to affect the release volume. The higher the tailings solids density, the higher the estimated release volume. The sensitivity assessment indicated that within a reasonable range of tailings solids densities, this failure mechanism still overestimated the liquefaction and overtopping failure mechanisms, but each of the sensitivities was in range of the slope instability failure mechanism reported release volume.

Table 13 Percent of total storage volume released for different approaches

Volume Estimation Approach	Liquefaction (%)	Overtopping (%)	Slope instability (%)
Empirical by total storage	29	30	28
Empirical by height	4	4	1
Geometric	17	5	1
Flowability	20	40	12
Reported percent of total storage volume released	11	9	13

While the tailings properties affect the release volume, this method is most sensitive to the pond volume. The slope instability failure mechanism had a pond volume of ≈ 3% of the total stored volume. The reason this method was within the range of estimating the reported release volume was likely because it best estimated the pond conditions at the time of failure.

Discussion

Each of the volume release estimates were compared with the reported release volume. The empirical method that estimates release volume as a function of total volume overestimated each of the three cases, while the empirical method that estimates it by height underestimated all three cases. The geometric approach provided the closest estimates for both the liquefaction and overtopping failure mechanisms, but underestimated the release volume for slope instability failure. The flowability approach overestimated the release volume by liquefaction and overtopping, but was within the range of the release volume for the slope instability failure mechanism. As discussed in the sensitivity assessment of the flowability approach, this method is highly dependent on the pond volume assumed. To accurately evaluate this method, an assessment should be considered for a historic failure where all parameters relating to the tailings properties and pond storage volume are known. The percentage of total storage volume released using the different methods for the various failure modes are summarized in Table 13. While it appears that the volume estimation methods do not accurately estimate the release volume, this is not necessarily the case. Additional historic failures would be required to definitively compare methods. Additional studies should be undertaken to correlate the release volume estimation methods to failure mechanisms, dam characteristics, and tailings properties.

Each of the methods are highly variable and depend on the input parameters. For both empirical methods, the total storage volume and dam height are generally known, so the level of uncertainty is reduced. However, the effects of the regressions when the historic data are separated by factors such as dam height, tailings rheology, and pond size should be considered. The flowability method is extremely dependent on pond size, and during normal operations of a TSF, this can be highly variable. If all other parameters are well defined (tailings solids density, etc.), sensitivity analyses should be undertaken to achieve various release volumes within reasonable operational ranges of the pond to be incorporated in a TDBA. This holds true when estimating the release volume (using any method), and it is recommended that a range of release volumes that consider variability in operating conditions of the TSF be considered and that a TDBA sensitivity analysis be undertaken within a bound of these volumes.

Tailings Dam Breach Release Hydrograph

Breach Parameters: An Overview

It is reasonable to assume that concrete dams fail completely and instantaneously; however, this not the case with earthen dams. As such, breach geometry and breach formation time are critical to the results of a TDBA. Many flood routing software programs incorporate built-in functionality to estimate the breach hydrograph (peak discharge and time to peak discharge). These breach hydrographs are estimated within the software using various empirical and geometric approaches, most of which require a set of breach parameters (Gee 2010):

Shape and progression: side slopes, breach height, and breach width.

Development time: critical breach development time.

Breach Side Slopes

The typical cross-section of a dam breach is a triangle or parallelogram, with the smaller edge at the breach bottom (breach bottom width). The breach side slope affects the overall size of the breach, which in turn affects the outflow hydrograph. Von Thun and Gillette (1990) and Dewey and Gillette (1993) proposed a value of the horizontal component of the breach slope (z) of 1, except for dams with cohesive shells or very cohesive cores, for which a value of 1/2 or 1/3 may be more appropriate. Froehlich (1995) suggests assuming z of 1.4 for overtopping and 0.9 for other failures. However, he found that the average side slope of 63 cases studied was 1.0. FERC (1987) suggested using a value of z between 0.25 and 1 for engineered and compacted dams and between 1 and 2 for non-engineered, slag, or refuse dams, using a larger area for weaker dams.

Breach Height

The breach height (h_b) is defined by Eq. (5), and presented in Fig. 6:

$$h_b = HDE - BME \tag{5}$$

where: HDE = top elevation of breach, m, and BME = breach invert elevation, m.

The top elevation of the breach and the breach invert elevation vary depending on the failure mechanism and physical constraints around the dam. For an overtopping failure, the top elevation of the breach is typically the crest elevation of the dam and the breach invert is the downstream toe elevation of the dam. The top elevation for piping (internal erosion) failures is typically taken as the water/tailings surface elevation. Piping (internal erosion) can occur at any elevation within the dam cross-section; thus, the breach invert elevation is often considered in the sensitivity analysis for the breach parameters to quantify the uncertainties.

Breach Width

The breach width is often computed as the average breach width (B), and this value is converted to the breach bottom width (BBW) as this is the input often required to compute the breach hydrograph. The breach bottom width is calculated as a geometrical relationship with the breach height (h_b) and breach side slope (z) (Eqs. 6 and 7):

$$BBW = B - zh_b \tag{6}$$

$$0.25 \leq z \leq 1 \tag{7}$$

The empirical formulas used to compute the breach width range are described under available regressions, below. The range of values is computed using formulas that refer to the dam construction material and failure mechanism.

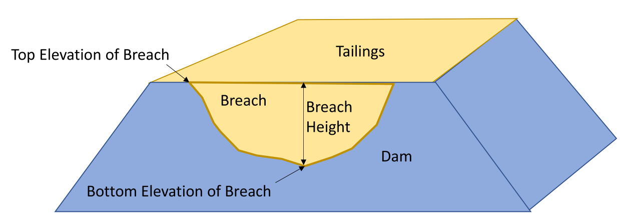


Fig. 6 Schematic view of breach opening

Development Time

A dam breach is not normally instantaneous, as two distinct phases with different mechanics and rates of erosion typically occur: breach initiation and breach development. During the breach initiation, the dam has not yet failed, and a limited outflow is produced. For overtopping failures, a small flow may occur over the dam crest, whereas for piping (internal erosion) failure, the initial flow may be observed through the developing pipe or seepage channel. This process may be used as a warning, and it may be possible to stop the breach from occurring. The breach development time, sometimes referred to as breach formation time, is characterized by an outflow and erosion that increases rapidly. The empirical formulas used to compute the breach development time range are described below. The range of values is computed using formulas that are relevant to the dam construction material and failure.

Available Regressions

There are several regression equations available for estimating the breach parameters (for both the shape progression and breach development time). These regressions have been estimated based on historic failures for water storage dams. There are limitations when applying these regressions to tailings dam failures, mostly related to the effect of adopting the regressions to estimate a peak outflow of a discharge with a sediment concentration, rather than simply water. A summary of some of the key available regression equations for estimating breach parameters (and corresponding breach hydrographs) are presented in Table 14.

Comparison of Software for Breach Hydrographs

Earlier in this paper, a list of available software programs that have built-in functionality for estimating the breach hydrograph for a given release volume was discussed. This section compares how well three of them estimated the breach hydrograph for a given case.

An important note in selecting the proper application to obtain a breach hydrograph is the immediate downstream topographic condition of the breach location, as it affects the breach hydrograph shape and peak. The *immediate downstream* of a breach location is the distance from the breach point to about 100 m (for a channel with gentle slope) to 300 m (for a channel with steep slope), where the backwater effect may change the breach hydrograph characteristics. To quantify the effect of the downstream condition on the breach hydrograph, three models were run with similar breach parameters and downstream topographic conditions: HEC-RAS 2D, FLDWAV, and HEC-HMS. As shown in Fig. 7, HEC-RAS 2D and FLDWAV modeled the breach hydrographs almost the same, especially for its peak and shape. However, the HEC-HMS model, which is a parametric model and does not include the downstream condition of the breach location, modeled a lower peak and wider hydrograph than the other two models. Volumes estimated by all three hydrographs were similar in this exercise. Similar volumes but different hydrograph peaks and shapes can affect the downstream inundation, particularly the flood arrival times, and thus the consequence of a failure. HEC-HMS would be considered the easiest in terms of model setup and execution, while FLDWAV requires more time to set up and run the model.

Tailings Dam Breach Flood Hydrograph Routing Downstream

Routing breach hydrographs along the flood flow-path downstream of a breach location is also a challenging task when conducting TDBAs. Developed computational fluid dynamics (CFD) models are well understood for Newtonian fluids (i.e. water dams), but are not understood as well for non-Newtonian fluids (i.e. tailings dams). The computational applications that are commonly used in TDBAs are briefly summarized below.

FLO-2D was created by Jimmy S. O'Brien for the Federal Emergency Management Agency in 1989, and was discussed by O'Brien and Julien (1993). The model predicts flood hazards, mudflows, and debris flows over alluvial fans using three different systems. A uniform grid is used to describe the floodplain topography, so the model can simulate interactive flood or mudflow routing between the channel, street, and floodplain flow. Both clear water and sediment flow flooding conditions are modeled using a quadratic, rheological model. The overall model is based on a grid system. Each sector in the grid is given a location, an elevation, and a roughness factor, and an also be given flow reduction factors by the user (e.g. in residential areas with buildings). Once these values are placed in the grid, flow is routed through it. Discharge is predicted using an estimated depth of flow for each sector, which is then computed and summed across all four boundaries of the floodplain. The model assumes that flow during the time step is steady, with hydrostatic pressure distributions, reasonably uniform cross-section shape and roughness, and single input values for each grid sector. The accuracy of the prediction is governed by the grid density. Use of FLO-2D to model dam breaches is reported by Breitkreutz (2011).

DAN3D is a model used to predict the run-out analysis of extremely high velocity landslides, including rock and debris avalanches. The model is an extension of DAN, a one-dimensional software that relies on a meshless, Lagrangian model that considers variability in the flow path. It allows

Table 14 Summary of breach parameter regression

Reference	Parameter	Piping	Overtopping	Note
USBR (1988)	Average breach width	$B = 3h_b$		Estimated using the Simplified Dam-Break (SMPDBK) model. The equation provided for the average breach width is only applicable for earthen dams and provides an upper bound value for the breach width including a factor of safety. h_b is the breach height
	Time to failure	$t_f = 0.011B$		
FERC (1987)	Average breach width	$B = 2h_b \; to \; 4h_b$ $h_b \le B \le 5h_b$		For earthen or rockfill dams
Fread (2001)	Average breach width	$B = 9.5 * 0.7\left(V_w h_w\right)^{0.25}$	$B = 9.5 * 1.0\left(V_w h_w\right)^{0.25}$	Statistically based on 60 historical dam failures reported by Froehlich (1987, 1995) and MacDonald and Langridge-Monopolis (1984) V_w is in acre-ft and h_w in ft
	Time to failure	$t_f = 0.3V_w^{0.53}h_b^{-0.90}$		
Froehlich (1995)	Average breach width	$B = 0.18034V_w^{0.32}h_b^{0.19}$	$B = 0.2524V_w^{0.32}h_b^{0.19}$	Improving previous regressive studies with additional historical dam breach cases, Froehlich (1995) proposed formulas for average breach width based on the failure mechanism V_w is volume of water above breach invert elevation at time of breach, m^3
	Time to failure	$t_f = 0.00254V_w^{0.55}h_b^{-0.90}$		
Froehlich (2008)	Average breach width	$B = 0.27V_w^{0.32}h_b^{0.04}$	$B = 0.351V_w^{0.32}h_b^{0.04}$	V_w is volume of water above breach invert elevation at time of breach, m^3
	Time to failure	$t_f = 63.2\sqrt{\frac{V_w}{gh_b^2}}$		
MacDonald and Langridge-Monopolis (1984)	Time to failure	$t_f = 0.0179\left(V_{er}\right)^{0.364}$ $V_{er} = 0.0261\left(V_{out}h_w\right)^{0.736}$		t_f is the breach formation time, hr V_{er} is the volume of embankment material eroded, m^3 V_{out} is the volume of water discharged through breach (initial storage and inflow during failure), m^3 Developed from the upper envelope of 42 case studies. The formula is based on the volume of the dam that will be eroded. Rockfill dams and earthfill dams with erosion-resistant core are considered to be non-earthfill
Von Thun and Gillette (1990)	Average breach width	$B = 2.5h_w + C_b$ $C_b = \begin{cases} 6.1 & V < 1.23 \times 10^6 \\ 18.3 & 1.23 \times 10^6 \le V < 6.17 \times 10^6 \\ 42.7 & 6.17 \times 10^6 \le V < 1.23 \times 10^7 \\ 54.9 & V > 1.23 \times 10^6 \end{cases}$		h_w is the height of water, m, and V is the reservoir capacity, m^3
	Time to failure	$t_f = 0.020h_w + 0.25 \; [erosion\ resistant]$ $t_f = 0.015h_w \; [easily\ erodible]$		A relationship for the development time based on the height of water and the type of dam (erosion resistant or easily erodible). The erodibility of the failed dams studied was not classified; rather, the erosion resistant and easily erodible category represent the lower and upper envelope of their predictive equation (USBR 2014)
Xu and Zhang (2009)	Average breach width	$Y_3 = \frac{B}{h_b}$		Y_3 and Y_5 are dimensionless parameters and are calculated using a multivariable regression Where $t_r = 1h$
	Time to failure	$Y_5 = \frac{t_f}{t_r}$		

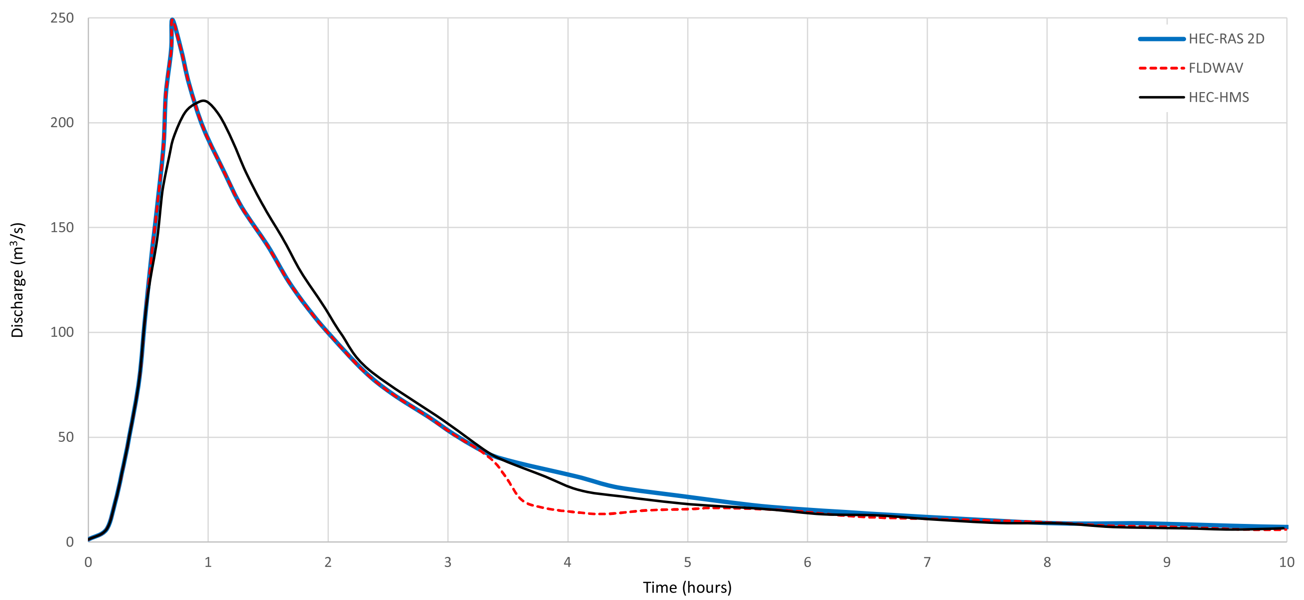


Fig. 7 Comparison of breach hydrograph resulted from HEC-RAS 2D, FLDWAV and HEC-HMS

the use of non-Newtonian flow, so tailings can be more accurately modeled. DAN3D is based on a two-dimensional Lagrangian solution of unsteady flow over three-dimensional surfaces. Like its two-dimensional version, DAN, it can interpolate and thereby model the entire surface, and in this case, the environment is smoothed in three dimensions. The model solves depth-averaged equations for an equivalent flow, which results from simple rheological relationships that are acquired through back calculations of real landslide analysis. DAN3D has the following key features: (1) simulation of flow over complex three-dimensional terrain without the need to input predictive flow paths or outcomes, (2) prediction of internal stresses resulting from three-dimensional deformation of material with internal shear stresses taking into account strain, anisotropic, and non-hydrostatic conditions, (3) prediction of the transfer of mass and momentum by taking into account entrainment of path material, and (4) ease of use.

FLOW-3D is a three-dimensional (3D) model that uses the finite volume method to spatially discretize the Navier Stokes (NS) equations throughout a given domain. The NS equations that describe the motion of viscous fluids are solved by numerical approximation using the finite volume method for each individual cell of the model. For dam breach analysis, FLOW-3D is often used in a two-dimensional (mostly due to long reach of flood routing downstream of the breach location) model (i.e. shallow water mode). In this case, FLOW-3D would not be able to account for erosion of channels downstream. Treating the release volume as a homogenous mixture of tailings and water that does not change along floodway is another major limitation of FLOW-3D.

Derivation of Equations

A Newtonian type of fluid is often considered as an isotropic homogeneous mixture that follows a linear relationship for viscosity. Newtonian fluid flow can be described using the NS equations in Eqs. (8–11) below.

Continuity equation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{8}$$

Momentum equations:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=-\frac{1}{\rho_0}\frac{\partial P}{\partial x}+\frac{\partial}{\partial x}\left(\upsilon_{eff}\left(\frac{\partial u}{\partial x}\right)\right)+\frac{\partial}{\partial y}\left(\upsilon_{eff}\left(\frac{\partial u}{\partial y}\right)\right)+\frac{\partial}{\partial z}\left(\upsilon_{eff}\left(\frac{\partial u}{\partial z}\right)\right) \tag{9}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=-\frac{1}{\rho_0}\frac{\partial P}{\partial y}+\frac{\partial}{\partial x}\left(\upsilon_{eff}\left(\frac{\partial v}{\partial x}\right)\right)+\frac{\partial}{\partial y}\left(\upsilon_{eff}\left(\frac{\partial v}{\partial y}\right)\right)+\frac{\partial}{\partial z}\left(\upsilon_{eff}\left(\frac{\partial v}{\partial z}\right)\right)-g\frac{\rho-\rho_0}{\rho_0} \tag{10}$$

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=-\frac{1}{\rho_0}\frac{\partial P}{\partial z}+\frac{\partial}{\partial x}\left(\upsilon_{eff}\left(\frac{\partial w}{\partial x}\right)\right)+\frac{\partial}{\partial y}\left(\upsilon_{eff}\left(\frac{\partial w}{\partial y}\right)\right)+\frac{\partial}{\partial z}\left(\upsilon_{eff}\left(\frac{\partial w}{\partial z}\right)\right) \tag{11}$$

where u, v, and w are the mean velocity components in the x, y, and z directions, respectively, t is the time, P is the mean fluid pressure, υ_{eff} represents the effective kinematic viscosity ($\upsilon_{eff}=\upsilon_t+\upsilon$), υ_t is the turbulent kinematic viscosity, υ is the kinematic viscosity, g is the gravity acceleration. In the above equations (considering only the x-direction), the terms can be explained as:

$$\text{Inertial Forces}: u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z} \tag{12}$$

$$\text{Pressure Forces}: \frac{1}{\rho_0}\frac{\partial P}{\partial x} \tag{13}$$

$$\text{Viscous Forces}: \frac{\partial}{\partial x}\left(\upsilon_{eff}\left(\frac{\partial u}{\partial x}\right)\right)+\frac{\partial}{\partial y}\left(\upsilon_{eff}\left(\frac{\partial u}{\partial y}\right)\right)+\frac{\partial}{\partial z}\left(\upsilon_{eff}\left(\frac{\partial u}{\partial z}\right)\right) \tag{14}$$

The above equations describe fluid motion in three dimensions. However, when horizontal length scales are much larger than the vertical length scales, the fluid motion can be described in two dimensions. These models are often called depth-averaged models.

Unlike the equations above, kinematic viscosity is not constant and thus sets of equations become more complex in non-Newtonian fluids. Different viscosity relationship exists for non-Newtonian fluids, as shown in Fig. 8. A non-Newtonian model that incorporates only the Bingham stresses and ignores the inertial stresses assumes that the simulated flow is dominated by viscous stresses. This assumption may not be universally appropriate as some mudflows are very turbulent with velocities sometimes reaching as high as 10 m/s. Even mudflows with solid concentrations of up to 40% by volume can be highly turbulent (O'Brien 1986). Depending on the fluid matrix properties, the viscosity and yield stresses for high sediment concentrations can still be relatively small compared to the turbulent stresses. If the flow is controlled primarily by the viscous stress, it will result in lower velocities. Conversely, if the viscosity and yield stresses are small, the turbulent stress will dominate, and the velocities will be higher.

There are several numerical models that are often used in industry for the tailings dam breach routing downstream of the breach, some of which were already summarized above. FLO-2D is one of the most common models being used in industry as it is the only model that accounts for the change in sediment concentration at each cell and time step in the model domain. FLO-2D solves two-dimensional (i.e. depth

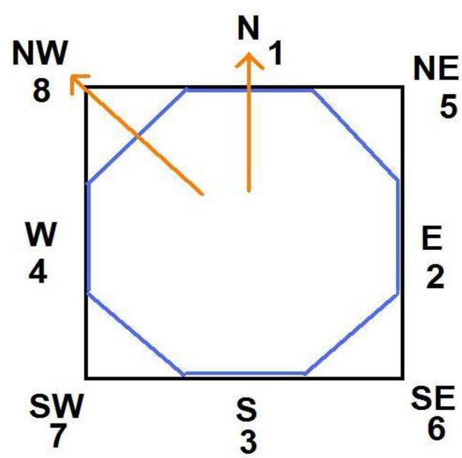
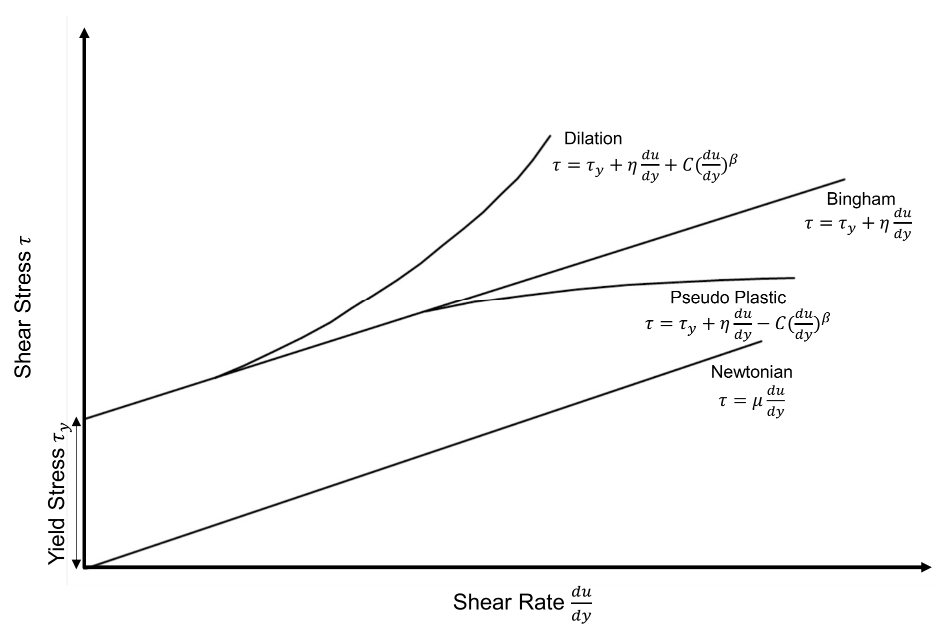


Fig. 9 Directions of flow calculations in FLO-2D (Source: FLO-2D)

Fig. 8 Shear stress versus shear rate for fluid deformation models (Sources: FLO-2D)



averaged) flow equations in eight directions, as shown in Fig. 9. Therefore, the deriving equations of FLO-2D will be reviewed briefly.

TDBAs are modeled in FLO-2D using its mudflow module for viscous hyper-concentrated sediment flows that have transient fluid properties. Unless a rheological analysis of the mudflow site material is available, the following empirical relationships can be used to compute viscosity and yield stress (Eqs. 15 and 16):

$$\mu = \alpha_1 e^{\beta_1 C_v} \tag{15}$$

$$\tau_y = \alpha_2 e^{\beta_2 C_v} \tag{16}$$

where αi and βi are empirical coefficients defined by laboratory experiment (O'Brien and Julien 1988). C_V is tailings concentration by volume and is calculated as per Eq. (17):

$$\mathrm{C_V} = \frac{\mathrm{C_W}\gamma_\mathrm{w}}{\gamma_\mathrm{s} - \mathrm{C_W}(\gamma_\mathrm{s} - \gamma_\mathrm{w})} \tag{17}$$

where C_W is the concentration by weight; γ_w is the specific weight of water; and γ_s is the specific weight of sediment. A quadratic model for rheology is used in FLO-2D to model viscosity and yield stresses as a function of tailings concentration. Total shear in FLO-2D is calculated as sum of dispersive shear stress, viscous shear stress, turbulent shear stress, and yield stress, as equated by Eq. (18):

$$\tau = \tau_\mathrm{y} + \tau_\mathrm{v} + \tau_\mathrm{t} + \tau_\mathrm{d} \tag{18}$$

O'Brien and Julien (1985), using the model for rheology, converted the above equation to Eq. (19):

$$\tau = \tau_\mathrm{y} + \mu\left(\frac{\mathrm{dv}}{\mathrm{dy}}\right) + \mathrm{C}\left(\frac{\mathrm{dv}}{\mathrm{dy}}\right)^2 \tag{19}$$

where C is the inertial stress coefficient.

To define all the shear stress terms for use in the FLO-2D model, the work of Meyer-Peter and Müller (1948) and Einstein (1950) was adopted. The shear stress relationship is depth integrated and rewritten in the following form as a dimensionless slope (Eq. 20):

$$S_f = S_y + S_v + S_{td} \tag{20}$$

where the total friction slope $\mathrm{S_f}$ is the sum of the yield slope $\mathrm{S_y}$, the viscous slope $\mathrm{S_v}$, and the turbulent-dispersive slope $\mathrm{S_{td}}$. The individual slope terms are calculated following the work of O'Brien and Julien (1993), as Eqs. (21–23).

$$\mathrm{S_y} = \frac{\tau_\mathrm{y}}{\rho g \mathrm{h}} \tag{21}$$

$$\mathrm{S_v} = \frac{\mathrm{K}\mu\mathrm{V}}{8\rho g \mathrm{h}^2} \tag{22}$$

$$\mathrm{S_{td}} = \frac{\mathrm{n_{td}^2 V^2}}{\mathrm{h}^{\frac{4}{3}}} \tag{23}$$

where K is the resistance factor; and $\mathrm{n_{td}}$ is the modified manning's coefficient. The modified Manning's n can be calculated as an exponential function of the volumetric concentration of sediment.

Numerical Test Example and Discussion

Based on the equations presented above and comparing them with derived equations of other CFD models (not presented here), a detailed modeling exercise was performed to quantify the differences in the inundation extents between the numerical models. FLO-2D and FLOW-3D models were used to perform these analyses. Identical input data and numerical details were used, as summarized in Table 15.

In the modeling comparison, the runout extent was larger under the same conditions when modeled using FLO-2D than with FLOW-3D. As mentioned earlier, FLOW-3D does not account for changes in sediment concentration, and the fluid paste would move downstream as a uniform and homogeneous mixture. This is the reason that the runout extent predicted by FLO-2D is larger (Fig. 10). It seems that the runout extent estimated by FLO-2D often exceeds that of FLOW-3D, regardless of the terrain and rheology parameters of the tailings when the fluid is in the range of mud flood and mud flow (i.e. $C_V = 20–55\%$), based on Fig. 11 (CDA 2020). FLO-2D and FLOW-3D results showed almost the same inundation extents (not presented here) for the water flood (i.e. $C_V < 20\%$). The results deviate more in the case of higher sediment concentrations in the fluid. For the sake of brevity, only $C_v = 55\%$ was simulated using both models

Table 15 Input parameters used in FLO-2D and FLOW-3D models

Parameter	Cell size (m)	Courant number	DEM resolution (m)	Manning's n	Breach peak flow × 1000 (m^3/s)	Tailings viscosity (poises)	Tailings yield stress (dynes/cm^2)	Tailings specific gravity
Value	10	< 0.5	1	0.03–0.06	30	350	3300	2.65

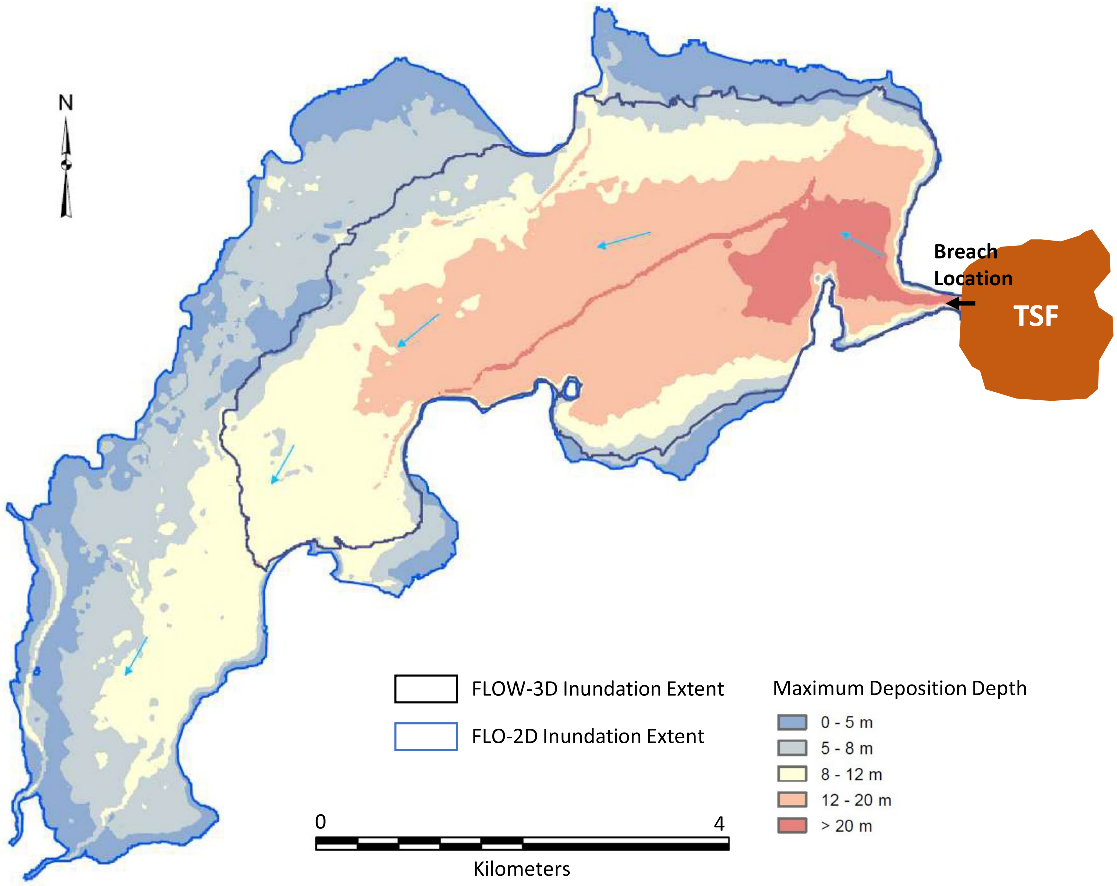


Fig. 10 Comparison of inundation extents between FLO-2D and FLOW-3D

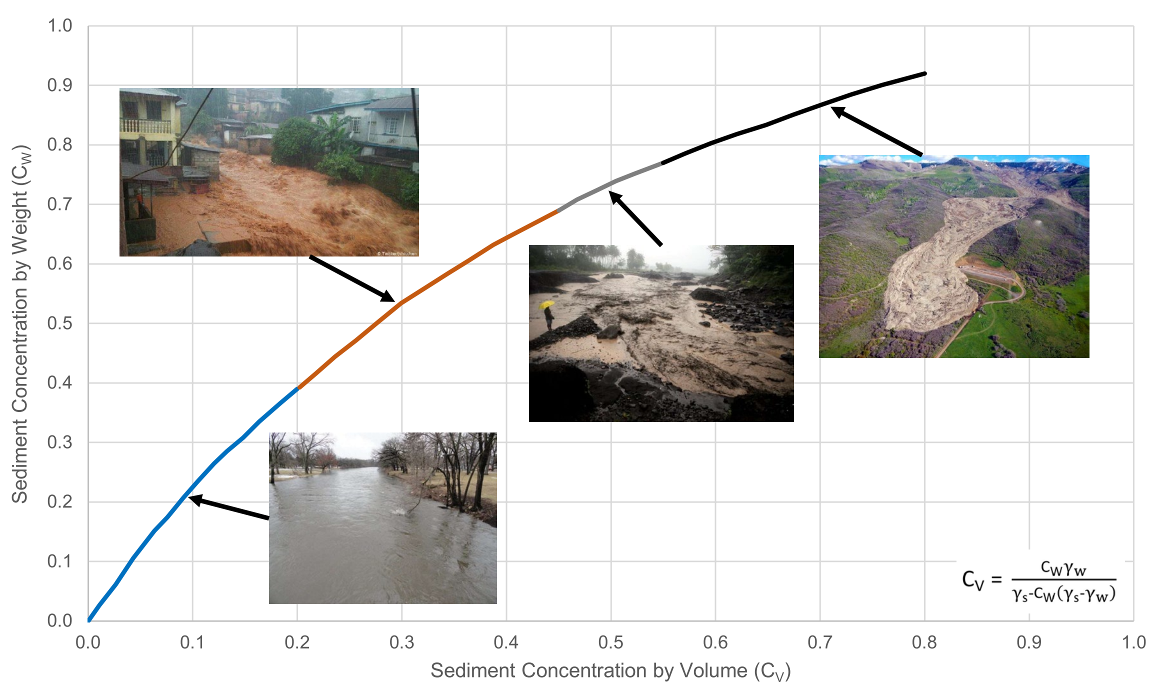


Fig. 11 Flow types as a function of sediment concentration (Source: CDA 2020)

(Fig. 10), as the maximum runout difference was observed in this case. No numerical test was performed for landslide cases in this study.

Other Important Factors and Research and Development

Missing Factors

It is well understood that there is a high uncertainty in TDBAs when estimating the tailings release volume and corresponding release velocity, inundation area, and inundation depth. Empirical equations derived using historic data are used to estimate the potential release volume, though no correlations have been developed to estimate the effects that different conditions have on the effectiveness of those release volumes. A missing, yet critical, factor in TDBAs is the development of methods to estimate the release volume, based on dam-specific conditions, such as tailings rheology, storage method (e.g. co-disposal, paste, or thickened tailings), tailings type (i.e. the mined material, such as copper tailings compared to iron tailings), dam height, dam construction method, and dam construction material.

This paper has discussed the available software for TDBA modeling. The software selected for modeling TDBAs is often governed by the geographical location of the dam. For example, in the Atacama Desert, with evaporation rates close to ten times higher than rainfall rates, in the absence of an operational pond at the dam, it is highly likely that the stored tailings are going to have a *Cv* in the range of mud flows or landslides, thus governing software selection. In regions with high seismic activity, the possibility that tailings could lose significant strength can make upstream raised facilities riskier, while this may not be a consideration in locations with low seismic activity. Across the globe, geographical conditions will determine what type of software and what type of scenarios are relevant. The software being used by industry today has been primarily developed for water storage facilities; modifications to cater software specifically for TDBAs would be a significant advancement in the field.

Climate change and the impact that it will have on future climatic conditions at dam sites must be considered in TDBAs. While there is no guideline or recommendation as to how this should be considered, the effect it may have on software selection and the credible scenarios chosen is important.

It is understood that topographic quality (resolution) can significantly affect a TDBA's results (Halliday and Arenas 2019). A high resolution and detailed topographic survey is able to capture key changes that occur across the terrain, while a low resolution survey may fail to capture key features, such as narrow valleys or small creeks, altering the results of a TDBA. Regulations and guidelines developed for TDBAs should include guidance on how to select an appropriate downstream topographic resolution for flood routing purposes, and emphasize the importance of mine owners investing in a downstream survey in sensitive environments, or those with drastically changing terrains.

Clearly, when considering TDBAs, one method does not suit all. Climatic and geographical region, dam type, tailings type, and the storage method all need to be considered for each specific location to get an accurate and realistic estimate of the release behavior, and to select the correct failure mechanisms and corresponding software. If a global method is adopted for undertaking TDBAs, specific guidance or flexibility must be given to accommodate the geographical differences that will arise for each site across the globe.

There is an inevitable uncertainty in future conditions. Engineers design dams not to fail, but the fact is they still do. It is impossible to accurately predict what human errors might occur at a facility in 20 years' time, how the weather might change, how processing schedules will be updated and modified, how to decide what a credible mode of failure is, and how to eliminate certain failure mechanisms given the magnitude of the life of a facility, and at what phase TDBAs should be undertaken. Should TDBAs be assessed under real-time conditions for emergency planning? How can mine owners and the public utilize inundation plans without a full understanding of the assumptions used to derive them? How can mine clients use the information correctly to develop and execute safe and accurate emergency response and preparedness plans? Guidelines should be developed to properly describe and regulate how frequently a TDBA should be undertaken.

The important missing factors for TDBAs are well understood, but it is critical that consultants and mine owners work together to reduce the uncertainty in TDBAs and ensure that all of the important factors have been identified and included in TSF designs. The quality of results obtained from a model can only be as good as the quality of information that goes into the model, and as such, consultants should ensure that the topographic quality used in assessments will not negatively influence modeling and mapping results.

Current Research and Development

Recent studies that have attempted to address some of the important missing factors are highlighted in this section.

Physical modeling of tailings dam breach (Queen's University, Canada): supported by the CanBreach (Canadian Tailings Dam Breach Research) Project (Walsh et al. 2019). This research is using an experimental modeling to

Fig. 12 Downstream oblique view of a typical dam during breach (Source: Walsh et al. 2019)

investigate tailings breach behavior, specifically the difference in the outflow hydrograph of water storage facilities compared with TSFs. A downstream image view of a typical dam during breach from the research is presented in Fig. 12.

Runout zone classification for the analysis of tailings flows (University of British Columbia, Canada): this research (Ghahramani et al. 2019) has established a runout zone classification method that could potentially be used for risk analysis. Future studies as part of the CanBreach project include building a database and investigating the effects of other attributes of the tailings and downstream topography, which could potentially be used to refine the area-volume empirical-statistical relationship.

*The risks of excess water on tailings facilities and its application to dam break studies (*Rourke and Luppnow 2015*)*: this study correlates the tailings release volume to pond size. The study has a strong correlation for five historic failures, but needs more data points to generate a realistic correlation.

A detailed methodology critique using historic failures for tailings release volume estimation (Halliday and Kheirkhah Gildeh): this study investigates the available methods for volume estimation and compares them against historic information. The study evaluates methods against failure mechanisms, dam heights, and storage volume to develop trends and correlations, and assesses if there are ways to estimate volumes that are more applicable for key features.

Conclusions and Recommendations

There is a common consensus that uncertainties exist in conducting TDBAs. Mine owners, local governments, national committees, consultants, and academia are working together to bridge the knowledge gaps. The following conclusions can be drawn from this paper:

- Historic failures educate consultants about potential failure and triggering mechanisms; detailed and site-specific failure mode analyses should be done before eliminating any failure mode as non-credible.
- Most volume release estimation methods have been developed based on the available historic dam failure data, a large portion of which was derived from water dam failures. These methods should be applied with caution by first assessing the applicability of the empirical equations to the TSF being assessed.
- Flowability release volume estimation is highly dependent on the supernatant pond volume; if the pond volume is larger, the release volume is also larger. Pond location should also be considered when undertaking an assessment using this method. If the pond is not near the dam, an alternative method should be adopted.
- Flowability release volume estimation requires an extensive knowledge of the tailings behaviour, whether these parameters are uncertain; when using this method for volume estimation, a sensitivity analysis that considers a range of parameters should be used to assess their effect on the release volume. Consequently, a reasonable bound for the release volume should be considered for the TDBA.
- Geometric release volume estimation is highly dependent on the breach slopes (of both the dam and the tailings) and downstream slope of the dam. Since the breach slopes are derived using historic and laboratory tests, sensitivities should be considered when adopting this method to develop a range of reasonable release volumes.
- The available methods for volume release estimation (within the limitations of this study) were unable to

accurately predict the release volume of historic failures. The actual release volume was captured within the range of values obtained from each of the four methods adopted. This suggests that until additional research has been undertaken, a variety of applicable methods should be used to capture a range of potential release volumes, using only those methods applicable to the site conditions.
- Estimation of peak discharge hydrographs can be implemented in several software programs but require certain assumptions by users. The software estimations should be cross-checked using the original historic data regressions to ensure the applicability of the method to the site-specific TSF.
- The breach hydrograph shape and peak are sensitive to the immediate downstream topographic conditions of the breach location, which should be noted when choosing a tool to obtain the breach hydrograph.
- Most software incorporates the overtopping and piping erosion failure mechanisms for developing a breach hydrograph, despite the most common failure mechanism (within the dataset used in this study) being liquefaction/earthquake-induced. The effect of this failure mechanism on the breach hydrograph should be further investigated, and potentially incorporated into software.
- Most CFD models are used in two dimensions (2D) for TDBA and mapping due to computational resources and costs, as well as the suitability of a depth-averaged assumption, because the horizontal extent of inundation is much greater than the vertical extent.
- Different CFD models are used by industry to complete the TDBA and mapping (FLOW-3D, FLO-2D, DAN 3D, etc.). Despite similar principal fluid mechanics equations, these programs solve different sets of equations for mud transport.
- A case study numerical comparison between FLO-2D and FLOW-3D showed a larger extent of inundation for the FLO-2D model for the same breach parameters, downstream conditions, and numerical details. This could be attributed to the sediment concentration change in the FLO-2D model; the FLOW-3D assumes a uniform homogenous fluid mixture when routing the breach fluid downstream, which could underestimate the loss of life and affect emergency preparedness plans, environmental clean-up processes, and consequence classifications.

The following recommendations can be taken from this paper:

- Practitioners should continue to keep up with current best practice industry standards and continue to incorporate state-of-the-art practice into their work.
- When undertaking a TDBA, all stakeholders (industry, regulating authorities, practitioners, and academia) should continue to work collaboratively on challenges for each specific project.
- When undertaking a TDBA, a range of release volumes should be considered to capture the variability in estimation methods.
- All stakeholders should support research efforts on the unknowns presented in this paper as well as other issues that may affect social, economic, and environmental aspects of public interest.

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