

Material selection using multi-criteria decision making methods for geomembranes

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Abstract: The present work aims to be a useful information about the novel MCDM methods applied on geomembranes in order to select the best alternative for construction. Qualitative and quantitative information will be used to do the best choice. AHP method is used to calculate the weight (importance) of the different criteria which uses the subjective information

to assign numeric values among 0 and 1, being better weights with values near to 1. COPRAS-G, OCRA, TOPSIS, VIKOR, EXPROM II and ORESTE methods are applied as MCDM methods in order to perform the more accurate selection using relevant information about the features of different materials. These methods use the weight obtained by AHP for ranking the best alternative, whose result contains subjective and objective information. Finally, all methods are standardised with a unique equations template, which makes it easier to understand all of them in subsequent applications.

Keywords: geomembranes; multi-criteria decision making; MCDM; qualitative; quantitative; standardised; weight.

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1 Introduction

The selection of a geomembrane liner depends upon the application in which it will be employed (Peggs and Thiel, 1999). High-density polyethylene (HDPE) geomembranes have been used exclusively in landfill applications, especially for bottom liners, because of their relatively high resistance to aggressive leachate components (August and Tatzky, 1984; Haxo and Haxo, 1989). In general, geomembranes consist of 96–97.5% of polyethylene resin, 2–3% of carbon black and 0.5–1.0% of other additives such as antioxidants and stabilisers (Hsuan, 2000). The resin used is basically a linear copolymer

polymerized using ethylene and α -olefin as a comonomer under low pressure with appropriate catalysts (Hsuan, 2000). The typical structure of the polyethylene consists of a monotonous progression of linked carbon atoms that are bonded to hydrogen (Rowe, 2001). The polyethylene in a geomembrane may take the form of crystal lamellae where the polyethylene chains are neatly folded and tightly packed, and looser amorphous layers where the chains or chain segments are pendants or disordered (Rowe, 2001). The lamellae are linked via tie molecules that start and end in the adjacent lamellae. Thus, some of the inherent properties of polyethylene depend on the packing structure and consequently, any changes in the molecules and/or molecular packing may alter the durability and the overall field performance of the geomembrane. But none of them has been developed a previous study of the selection of material for geomembranes. Usually, engineers and researchers use certain materials based on experience and other studies.

1.1 Material selection by MCDM

The selection of the most convenient material for a precise purpose is a crucial function in the design and development of products. Materials selection has become an important source of engineering processes because of economic, technological, environmental parameters (Tawancy et al., 2007; Jee and Kang, 2000). Materials influence product function, the life cycle of the product, who is going to use or produce it, usability, product personality, environment and costs in multiple, complex and not always the quantitative way. The improper selection of one material could negatively affect productivity, profitability, cost and image of an organisation because of the growing demands for extended producer responsibility (Tawancy et al., 2007; Jee and Kang, 2000). For this reason, the development of products and the success and competitiveness of manufacturing organisations also depends on the selected materials (Ashby et al., 2004; Çalışkan et al., 2013).

Material selection carried out several research processes that give off assessment methods to compare the behavior of elements according to their characteristic properties (density, yield strength (YS), specific heat (SH), cost, corrosion rate, thermal diffusivity, etc.) with efficiency indicators in order to select the best alternative for a given engineering application (Ashby et al., 2004). Thus, efforts need to be extended to identify those criteria that influence material selection for a given engineering application to eliminate unsuitable alternatives and select the most appropriate alternative using simple and logical method (Chatterjee et al., 2011).

Comparing candidate materials, ranking and choosing the best material is one of most important stages in material selection process. Multi-criteria decision making (MCDM) methods appear as an alternative in engineering design due to its adaptability for different applications. The MCDM methods can be broadly divided into two categories, as

- 1 multi-objective decision-making (MODM)
- 2 multi-attribute decision-making (MADM).

There are also several methods in each of the above-mentioned categories. Priority-based, outranking, preferential ranking, distance-based and mixed methods are some of the popular MCDM methods as applied for evaluating and selecting the most suitable materials for diverse engineering applications. In most MCDM methods a certain weight is assigned to each material requirement (which depends on its importance to the performance of the design). Assigning weight factor to each material property must be done with care to prevent bias or getting the answer you intended as.

Here are some engineering applications where MCDM have been regarded as selection tools, performed by Jahan et al. (2010) 'Material screening and choosing methods – a review', developed by Findik and Turan (2012) 'Evaluating the construction methods of cold-formed steel structures in reconstructing the areas damaged in natural crises, using the methods AHP and COPRAS-G', studied by Bitarafan et al. (2012) 'Materials selection for lighter wagon design with a weighted property index method', developed by Ashby et al. (2004) 'Material selection for the tool holder working under hard milling conditions using different multi criteria decision making methods'.

This paper solves the problem of selecting the material a geomembrane using recent mathematical tools and techniques for accurate ranking of the alternative materials for a given engineering application. In this paper, it has been studied the material decision for pipes and vessel of the reactor by four preference ranking-based MCDM methods, i.e., COPRAS-G, OCRA, TOPSIS, VIKOR and EXPROM2 methods have been implemented. The criteria weighting was performed by compromised weighting method composed of AHP and Entropy methods. For these methods, a list of all the possible choices from the best to the worst suitable materials is obtained, taking into account different material selection criteria.

2 Materials and methods

2.1 Definition of the decision making problem

In order to meet for the material selection, it has been identified the most important properties based on the bibliography (Peggs and Thiel, 1999; Apse, 1989). The most in one of the most important material property are considered to permeability (P), the highest permeability is desired for its service application. The second properties required is the cost (C) and durability. For the cost, low values of which are desired in order to provide a competitive advantage among manufacturers. In addition, higher pressures would be favourable. Durability is divided into water durability (WD) and UV durability (UVD). The third important properties are the mechanical properties of Young's modulus (YM), Shear modulus (SM) and YS. The fourth important properties are the thermal properties thermal conductivity (TC), SH and thermal expansion (TE). Among these eight criteria, the cost and TE, are non-beneficial properties. Seven alternatives for the geomembranes were taken into consideration shown in Table 1. The properties of the materials alternatives for geomembranes with their quantitative data are given in Tables 2 and 3, as well as their average values, were used.

Table 1 Catalogue of geomembranes

<i>No.</i>	<i>CES search name</i>	<i>Material</i>	<i>Acronym</i>
1	PVC (flexible, Shore A60)	Polyvinyl chloride	PVC
2	Chlorinated polyethylene (unreinforced)	Chlorinated polyethylene	CPE
3	Chlorosulphonated polyethylene (unreinforced)	Chlorosulphonated polyethylene	CSPE
4	Ethylene propylene (diene) (EPDM/EPM, unreinforced)	Ethylene propylene rubber	EPDM
5	PP (homopolymer, low flow)	Polypropylene	PP
6	PE-LLD (moulding and extrusion)	Linear low-density polyethylene	LLDPE
7	PE-HD (general purpose, moulding and extrusion)	High-density polyethylene	HDPE

2.2 Subscript notation for MCDM

Commonly, in an MCDM problem there a number of alternatives to be evaluated in a given set of criteria. Every alternative is assigned a score reflecting its relative performance on each of the criteria considered (Parkan and Wu, 2000). In this work data from Geomembranes must be standardised as the Table 4 in order to apply the corresponding MCDM methods as easy and fast as possible.

The alternatives are sorted out in the first column which contains m elements, for this work there are seven alternatives of geomembranes. The criteria took into account are positioned in the first row which contains n elements, in this particular case, are ten different criteria. It is important to assign the type of criteria to differentiating between input and output. Input in contrast with output refers to non-beneficial criteria such as cost in which the best alternative for the j^{th} criteria is the lower value. Output determines the beneficial criteria in which the best corresponds to the higher value.

The weights represent the importance of each j^{th} criteria, thus the problem will contain n weights which will be calculated with qualitative information by using the following methods. Finally, the batch assigned for the i^{th} alternative at the j^{th} criteria is determined by the $f_{i,j}$ value. The standardisation applied to geomembranes is shown in Table 5.

2.3 Criteria weighting

The criteria weights are calculated using a compromised weighting method, where the AHP method. The AHP method was developed by Saaty (1990) to model subjective decision-making processes based on multiple criteria in a hierarchical system. The method composes of three principles:

In order to identify the importance of every alternative in an application, each alternative has been assigned a value. The ranking is composed by three levels: (Saaty, 1990)

- 1 general objective
- 2 criteria for every alternative
- 3 alternatives to regard.

Table 2 Properties of geomembranes obtained from CESedupack2013

No.	Permeability O ₂ (P)		Cost (C)		Durability		Mechanical properties	
	Non-beneficial	Input	Non-beneficial	Input	Water (DW)	UV radiation (DUV)	Young's modulus (YM)	Shear modulus (SM)
					Beneficial	Output	Beneficial	Output
1	3,180.00	3,510.00	2.13	2.59	Excellent	Fair	0.0030	0.0035
2	6.57	16.42	3.29	3.87	Excellent	Good	0.0020	0.0030
3	180.00	190.00	5.18	6.10	Acceptable	Good	0.0029	0.0100
4	1,590.00	1,960.00	3.90	4.29	Excellent	Fair	0.0007	0.0017
5	58.30	99.70	1.95	2.14	Excellent	Poor	1.3400	1.5900
6	131.00	219.00	1.83	2.05	Excellent	Poor	0.2620	0.5170
7	49.80	69.40	1.76	1.94	Excellent	Fair	1.0700	1.0900
Weights	0.4691		0.2010		0.1005	0.1005	0.0287	0.0287
						0.2010		0.0862

Table 3 Continued properties of geomembranes obtained from CESedupack2013

No.	Mechanical properties		Thermal properties					
	Yield strength (YS)		Thermal conductivity (TC)		Specific heat (SH)		Thermal expansion (TE)	
	Beneficial	Output	Beneficial	Output	Beneficial	Output	Non-beneficial	Input
1	10.00	11.00	0.160	0.200	1,600	1,800	155	165
2	12.00	13.00	0.200	0.300	1,800	2,000	160	200
3	20.30	31.80	0.200	0.300	1,800	2,000	160	200
4	1.50	2.50	0.150	0.200	2,010	2,110	200	300
5	32.90	36.40	0.205	0.213	1,670	1,700	97.9	100
6	9.65	19.30	0.155	0.162	1,840	1,910	297	303
7	26.20	31.00	0.461	0.502	1,750	1,810	106	198
Weights	0.0287		0.0142		0.0142		0.0142	
	0.0862				0.0427			

Table 4 The standard matrix

Material	C_1 type	C_2 type	C_3 type	...	C_n type
A_1	$f_{1,1}$	$f_{1,2}$	$f_{1,3}$...	$f_{1,n}$
A_2	$f_{2,1}$	$f_{2,2}$	$f_{2,3}$...	$f_{2,n}$
A_3	$f_{3,1}$	$f_{3,2}$	$f_{3,3}$...	$f_{3,n}$
\vdots	\vdots	\vdots	\vdots	\ddots	\vdots
A_m	$f_{m,1}$	$f_{m,2}$	$f_{m,3}$...	$f_{m,n}$
Weights	w_1	w_2	w_3	...	w_n

Table 5 Properties of geomembranes standardised

Material	P input	C input	DW output	DUV output	YM output	SM output	YS output	TC output	SH output	TE output
1	3345.0	2.360	4	2	0.0033	0.0012	10.500	0.1800	1,700	160.00
2	11.5	3.580	4	3	0.0025	0.0013	12.500	0.2500	1,900	180.00
3	185.0	5.640	3	3	0.0065	0.0022	26.050	0.2500	1,900	180.00
4	1775.0	4.095	4	2	0.0012	0.0004	2.000	0.1750	2,060	250.00
5	79.0	2.045	4	1	1.4650	0.2881	34.650	0.2090	1,685	98.95
6	175.0	1.940	4	1	0.3895	0.1355	14.475	0.1585	1,875	300.00
7	59.6	1.850	4	2	1.0800	0.3805	28.600	0.4815	1,780	152.00
Weights	0.4691	0.2010	0.1005	0.1005	0.0287	0.0287	0.0287	0.0142	0.0142	0.0142

The weight of criteria respect to other is set in this section. To quantify each coefficient, it is required experience and knowledge of the application (Saaty, 1990) classified the importance parameters shown in Table 2. The relative importance of two criteria is rated using a scale with the digits 1, 3, 5, 7 and 9, where 1 denotes ‘equally important’, 3 for ‘slightly more important’, 5 for ‘strongly more important’, 7 for ‘demonstrably more important’ and 9 for ‘absolutely more important’. The values 2, 4, 6 and 8 are applied to differentiate slightly differing judgments. The comparison among n criteria is resumed in the matrix $A_{m \times n}$ with the global arrange in equation (1).

$$A = \begin{bmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,n} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m,1} & a_{m,2} & \cdots & a_{m,n} \end{bmatrix} \quad a_{i,i} = 1, a_{j,i} = \frac{1}{a_{i,j}}, a_{i,j} \neq 0 \quad (1)$$

Afterwards, from matrix A it is determined the relative priority among properties. The eigenvector w is the weight importance and it corresponds with the largest eigenvector (λ_{max}):

$$(A - \lambda_{max})w = 0 \quad (2)$$

The consistency of the results is resumed by the pairwise comparison of alternatives. Matrix A can be ranked as 1 and $\lambda_{max} = n$ (Bayazit, 2005). In order to ensure the consistency of the subjective perception and the accuracy of the results, it is necessary to distinguish the importance of alternatives among them. In equations (3) and (4) is shown the consistency indexes required to validate the results.

$$CI = \frac{\lambda_{max} - n}{n - 1} \quad (3)$$

$$CR = \frac{CI}{RI} \quad (4)$$

where

- n number of selection criteria
- RI random index
- CI consistency index
- CR consistency relationship.
- $\lambda_{max}(A)$ largest eigenvalue.

If CR should be greater than 0.1, otherwise, the importance coefficient (Saaty, 1990) has to be set again and CR recalculated. For further references about AHP calculator see AHP calculator-AHP-OS or go to the next url: https://bpmsg.com/academic/ahp_calc.php.

2.4 COPRAS-G method

COPRAS-G method is a MCDM method that applies grey numbers to evaluate several alternatives of an engineering application. The grey numbers are a section of the grey theory to confront insufficient or incomplete information. White number, grey number, and black number are the three classifications to distinguish the uncertainty level of information (Chatterjee et al., 2011).

Let the number $F = [\underline{f}, \bar{f}] = \{f | \underline{f} \leq f \leq \bar{f}\}$ and $f \in R$, where F has two real numbers, \underline{f} (the lower limit of F) and \bar{f} (the upper limit of F) is defined as follows (Chatterjee et al., 2011):

- White number: if $\underline{f} = \bar{f}$ then F has the complete information.
- Grey number: $F = [\underline{f}, \bar{f}]$ means insufficient and uncertain information.
- Black number: if $\underline{f} \rightarrow -\infty$ and $\bar{f} \rightarrow \infty$, then F has no meaningful information.

The COPRAS-G method uses a stepwise ranking and evaluating procedure of the alternatives whose values are expressed in intervals (grey numbers) (Chatterjee et al., 2011; Zolfani et al., 2012). Before to proceed with this method, it is important to do a slight modification to Table 4 in order to employ grey numbers or intervals. The modifications for the subscript notation table are shown below in Table 6.

Table 6 Decision matrix modified for intervals

Material	C_1 type	C_2 type	C_3 type	...	C_n type
A_1	$F_{1,1}$	$F_{1,2}$	$F_{1,3}$...	$F_{1,n}$
A_2	$F_{2,1}$	$F_{2,2}$	$F_{2,3}$...	$F_{2,n}$
A_3	$F_{3,1}$	$F_{3,2}$	$F_{3,3}$...	$F_{3,n}$
\vdots	\vdots	\vdots	\vdots	\ddots	\vdots
A_m	$F_{m,1}$	$F_{m,2}$	$F_{m,3}$...	$F_{m,n}$
Weights	w_1	w_2	w_3	...	w_n

Then, the procedure of applying COPRAS-G method is formulated by the following steps:

- 1 Constructing decision making matrix.

$$\begin{aligned}
 & \begin{bmatrix} F_{1,1} & F_{1,2} & \cdots & F_{1,n} \\ F_{2,1} & F_{2,2} & \cdots & F_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ F_{m,1} & F_{m,2} & \cdots & F_{m,n} \end{bmatrix} \\
 &= \begin{bmatrix} [\underline{f}_{1,1}, \bar{f}_{1,1}] & [\underline{f}_{1,2}, \bar{f}_{1,2}] & \cdots & [\underline{f}_{1,n}, \bar{f}_{1,n}] \\ [\underline{f}_{2,1}, \bar{f}_{2,1}] & [\underline{f}_{2,2}, \bar{f}_{2,2}] & \cdots & [\underline{f}_{2,n}, \bar{f}_{2,n}] \\ \vdots & \vdots & \ddots & \vdots \\ [\underline{f}_{m,1}, \bar{f}_{m,1}] & [\underline{f}_{m,2}, \bar{f}_{m,2}] & \cdots & [\underline{f}_{m,n}, \bar{f}_{m,n}] \end{bmatrix}; \quad \begin{matrix} i = 1, \dots, m \\ j = 1, \dots, n \end{matrix}
 \end{aligned}$$

- 2 Normalised values of decision making matrix are obtained by using equations (5) and (6).

- Normalisation for lower values of the j^{th} criteria for the i^{th} alternative.

$$\dot{f}_{i,j} = \frac{2\underline{f}_{i,j}}{\sum_{i=1}^m \underline{f}_{i,j} + \sum_{i=1}^m \bar{f}_{i,j}}; \quad \begin{matrix} i = 1, \dots, m \\ j = 1, \dots, n \end{matrix} \quad (5)$$

- Normalisation for upper values of the j^{th} criteria for the i^{th} alternative.

$$\dot{\bar{f}}_{i,j} = \frac{2\bar{f}_{i,j}}{\sum_{i=1}^m \underline{f}_{i,j} + \sum_{i=1}^m \bar{f}_{i,j}}; \quad \begin{matrix} i = 1, \dots, m \\ j = 1, \dots, n \end{matrix} \quad (6)$$

Normalisation of decision making matrix gives another matrix of the same length with values between and 0 and 1.

$$\begin{bmatrix} [\dot{\underline{f}}_{1,1}, \dot{\bar{f}}_{1,1}] & [\dot{\underline{f}}_{1,2}, \dot{\bar{f}}_{1,2}] & \cdots & [\dot{\underline{f}}_{1,n}, \dot{\bar{f}}_{1,n}] \\ [\dot{\underline{f}}_{2,1}, \dot{\bar{f}}_{2,1}] & [\dot{\underline{f}}_{2,2}, \dot{\bar{f}}_{2,2}] & \cdots & [\dot{\underline{f}}_{2,n}, \dot{\bar{f}}_{2,n}] \\ \vdots & \vdots & \ddots & \vdots \\ [\dot{\underline{f}}_{m,1}, \dot{\bar{f}}_{m,1}] & [\dot{\underline{f}}_{m,2}, \dot{\bar{f}}_{m,2}] & \cdots & [\dot{\underline{f}}_{m,n}, \dot{\bar{f}}_{m,n}] \end{bmatrix}; \quad \begin{matrix} i = 1, \dots, m \\ j = 1, \dots, n \end{matrix}$$

- 3 Calculation of the weighted normalised decision matrix with the following equations.

$$\ddot{f}_{i,j} = \dot{f}_{i,j} \cdot w_j \quad ; \quad \begin{array}{l} i = 1, \dots, m \\ j = 1, \dots, n \end{array} \quad (7)$$

$$\ddot{f}_{i,j} = \dot{f}_{i,j} \cdot w_j \quad ; \quad \begin{array}{l} i = 1, \dots, m \\ j = 1, \dots, n \end{array} \quad (8)$$

Equations (7) and (8) shows how to build the weighted normalised decision matrix where w_j represents the weight for each j^{th} criteria obtained from AHP method. A new matrix as below is obtained with which the last steps will be executed.

$$\begin{bmatrix} \left[\begin{array}{ccc} \ddot{f}_{1,1}, \ddot{f}_{1,1} \\ \ddot{f}_{2,1}, \ddot{f}_{2,1} \\ \vdots \\ \ddot{f}_{m,1}, \ddot{f}_{m,1} \end{array} \right] & \left[\begin{array}{ccc} \ddot{f}_{1,2}, \ddot{f}_{1,2} \\ \ddot{f}_{2,2}, \ddot{f}_{2,2} \\ \vdots \\ \ddot{f}_{m,2}, \ddot{f}_{m,2} \end{array} \right] & \cdots & \left[\begin{array}{ccc} \ddot{f}_{1,n}, \ddot{f}_{1,n} \\ \ddot{f}_{2,n}, \ddot{f}_{2,n} \\ \vdots \\ \ddot{f}_{m,n}, \ddot{f}_{m,n} \end{array} \right] \\ \vdots & \vdots & \ddots & \vdots \end{bmatrix} ; \quad \begin{array}{l} i = 1, \dots, m \\ j = 1, \dots, n \end{array}$$

- 4 Two sums are calculated from the previous weighted normalised decision matrix. In this step, it is important to differentiate among input and output criteria.

- For output (beneficial) criteria, equation (9) is applied.

$$R_i = \frac{1}{2} \sum_{j=1}^k (\ddot{f}_{i,j} + \ddot{f}_{i,j}) \quad ; \quad i = 1, \dots, m \quad (9)$$

- For input (non-beneficial) criteria, equation (10) is applied.

$$P_i = \frac{1}{2} \sum_{j=k+1}^n (\ddot{f}_{i,j} + \ddot{f}_{i,j}) \quad ; \quad i = 1, \dots, m \quad (10)$$

Output criteria go from $j = 1$ representing the first to $j = k$ representing the last output criteria. Similarly, input criteria go from $j = k + 1$ representing the first to $j = n$ representing the last input criteria.

- 5 Determine the minimum value of R_i .

$$R^- = \min_{h=1, \dots, m} (R_h) \quad (11)$$

- 6 Calculating the relative significance of each alternative with the following expression.

$$Q_i = P_i + \frac{R^- \sum_{i=1}^m R_i}{R_i \sum_{i=1}^m \frac{R^-}{R_i}} \quad ; \quad i = 1, \dots, m \quad (12)$$

- 7 Determine the quantitative utility for the i^{th} alternative. The formula (13) is as follows:

$$N_i = \frac{Q_i}{\max_{h=1, \dots, m} (Q_h)} \cdot 100\% \quad ; \quad i = 1, \dots, m \quad (13)$$

- 8 The ranking will be performed in a decreasing order in which the best alternative will have the highest N_i value and the worst alternative will have the lower N_i value.

2.5 OCRA method

The OCRA method was developed to measure the relative performance of a set of production units, where resources are consumed to create value-added outputs. OCRA uses an intuitive method for incorporating the decision maker's preferences about the relative importance of the criteria. The general OCRA procedure is described as below (Parkan and Wu, 1999, 2000). In order to apply this MCDM method, it is crucial to apply the standardised matrix of Table 4 to the data.

- 1 Computation of input and output criteria. First, it is necessary to determine the maximum and minimum scores with equations (14) and (15) for each j^{th} criteria individually without taking into account if the criteria are input (non-beneficial) or output (beneficial). The counter $h = 1, \dots, m$ is used to do not confuse with the counters of the equations.

$$f_j^+ = \max_{h=1, \dots, m} f_{h,j} \quad ; \quad j = 1, \dots, n \quad (14)$$

$$f_j^- = \min_{h=1, \dots, m} f_{h,j} \quad ; \quad j = 1, \dots, n \quad (15)$$

- 2 From this step, input and output criteria are computed separately. Equations (16) and (17) are used with output criteria only. The different output criteria begin in 1 and ends in k .

$$o_i = \sum_{j=1}^k w_j \frac{f_{i,j} - f_j^-}{f_j^-} \quad ; \quad i = 1, \dots, m \quad (16)$$

$$O_i = o_i - \min_{h=1, \dots, m} o_h \quad ; \quad i = 1, \dots, m \quad (17)$$

- 3 In this step, we are only concerned with the scores that the various alternatives receive for the input criteria involved and omit the outputs. With the following equations (18) and (19) the computation is performed. The input criteria begins in $k + 1$ and finishes in n .

$$i_i = \sum_{j=k+1}^n w_j \frac{f_j^+ - f_{i,j}}{f_j^-} \quad ; \quad i = 1, \dots, m \quad (18)$$

$$I_i = i_i - \min_{h=1, \dots, m} i_h \quad ; \quad i = 1, \dots, m \quad (19)$$

- 4 Computation of the overall preference ratings. The computations of output and input criteria performed previously are applied in the next equation (20).

$$P_i = (I_i + O_i) - \min_{h=1, \dots, m} (I_h + O_h) \quad ; \quad i = 1, \dots, m \quad (20)$$

- 5 Similarly to COPRAS-G method, the rating is gauged by the P_j rating. The alternative with the higher P_j score will be the best choice and the alternative with the lower P_j will be the worst election.

2.6 TOPSIS method

The basic idea of TOPSIS is that the best decision should be made to be closest to the ideal and farthest from the non-ideal (Turskis and Zavadskas, 2010). Such ideal and negative-ideal solutions are computed by considering the various alternatives. The highest percentage corresponds to the best alternative. The TOPSIS approach is structured by the following procedure (Turskis and Zavadskas, 2010; Opricovic and Tzeng, 2004).

- 1 Calculate the normalised decision matrix by using the standardised data as Table 4. The normalisation is done by equation (21).

$$\dot{f}_{i,j} = \frac{f_{i,j}}{\sqrt{\sum_{i=1}^m f_{i,j}^2}} \quad ; \quad \begin{array}{l} i = 1, \dots, m \\ j = 1, \dots, n \end{array} \quad (21)$$

- 2 Calculate the weighted normalised decision matrix with equation (22).

$$\ddot{f}_{i,j} = \dot{f}_{i,j} \cdot w_j \quad ; \quad \begin{array}{l} i = 1, \dots, m \\ j = 1, \dots, n \end{array} \quad (22)$$

- 3 Determine the ideal and non-ideal solutions for each j^{th} criteria individually by taking into account the type of criteria such as input and output. Data from the weighted normalised decision matrix is used for this step.

- Output criteria are treated by equation (23).

$$\begin{array}{l} \ddot{f}_j^+ = \max_{h=1, \dots, m} \ddot{f}_{h,j} \\ \ddot{f}_j^- = \min_{h=1, \dots, m} \ddot{f}_{h,j} \end{array} \quad ; \quad j = 1, \dots, k \quad (23)$$

- Input criteria are treated by equation (24).

$$\begin{array}{l} \ddot{f}_j^+ = \min_{h=1, \dots, m} \ddot{f}_{h,j} \\ \ddot{f}_j^- = \max_{h=1, \dots, m} \ddot{f}_{h,j} \end{array} \quad ; \quad j = k + 1, \dots, n \quad (24)$$

Output criteria go from $j = 1$ representing the first to $j = k$ representing the last output criteria. Similarly, input criteria go from $j = k + 1$ representing the first to $j = n$ representing the last input criteria.

- 4 The distance between the ideal and non-ideal solutions are quantified. The two Euclidean distances for each alternative are computed as given by equations (25) and (26).

$$D_i^+ = \sqrt{\sum_{j=1}^n (\ddot{f}_{i,j} - \ddot{f}_j^{+2})} \quad ; \quad i = 1, \dots, m \quad (25)$$

$$D_i^- = \sqrt{\sum_{j=1}^n (\ddot{f}_{i,j} - \ddot{f}_j^{-2})} \quad ; \quad i = 1, \dots, m \quad (26)$$

- 5 The relative closeness is computed by using the equation (27).

$$C_i = \frac{D_i^-}{D_i^+ + D_i^-} \quad ; \quad i = 1, \dots, m \quad (27)$$

- 6 The final ranking is made in descending order like the previous methods.

2.7 VIKOR method

The VIKOR method was originally developed to solve decision problems with conflicting and different criteria, assuming that compromise is acceptable for conflict resolution, the decision maker wants a solution that is the closest to the ideal, and the alternatives are evaluated according to all established criteria. The VIKOR method considers the closeness to the ideal solution in order to rank the alternatives (Khorshidi and Hassani, 2013). The value closest to zero corresponds to the best alternative. This method is standardised with Table 4, thus the data must be computed previously in order to apply the following methods (Khorshidi and Hassani, 2013; Opricovic and Tzeng, 2004).

- 1 Select the maximum magnitude and minimum magnitude values of all criterion function with taking into account the type (input or output).

- Output criteria are treated by equation (28).

$$\begin{aligned} f_j^+ &= \max_{h=1, \dots, m} f_{h,j} \\ f_j^- &= \min_{h=1, \dots, m} f_{h,j} \end{aligned} \quad ; \quad j = 1, \dots, k \quad (28)$$

- Input criteria are treated by equation (29).

$$\begin{aligned} f_j^+ &= \min_{h=1, \dots, m} f_{h,j} \\ f_j^- &= \max_{h=1, \dots, m} f_{h,j} \end{aligned} \quad ; \quad j = k + 1, \dots, n \quad (29)$$

Output criteria go from $j = 1$ representing the first to $j = k$ representing the last output criteria. Similarly, input criteria go from $j = k + 1$ representing the first to $j = n$ representing the last input criteria.

- 2 Compute the values S_i and R_i by the equations (30) and (31).

$$S_i = \sum_{j=1}^n w_j \frac{f_j^+ - f_{i,j}}{f_j^+ - f_j^-} \quad ; \quad i = 1, \dots, m \quad (30)$$

$$R_i = \max_{j=1, \dots, n} \left(w_j \frac{f_j^+ - f_{i,j}}{f_j^+ - f_j^-} \right) \quad ; \quad i = 1, \dots, m \quad (31)$$

where w_j are the corresponding weights for the j^{th} criteria.

- 3 Compute the maximum and minimum values for S_i and R_i when $i = 1, \dots, m$ with the following equations (32) and (33).

$$\begin{aligned} S^+ &= \max_{i=1, \dots, m} S_i \\ S^- &= \min_{i=1, \dots, m} S_i \end{aligned} \quad (32)$$

$$\begin{aligned} R^+ &= \max_{i=1, \dots, m} R_i \\ R^- &= \min_{i=1, \dots, m} R_i \end{aligned} \quad (33)$$

- 4 Compute the values Q_i with the following equation.

$$Q_i = v \frac{(S_i - S^+)}{(S^- - S^+)} + (1 - v) \frac{(R_i - R^+)}{(R^- - R^+)} \quad ; \quad i = 1, \dots, m \quad (34)$$

v is used as the weight of the strategy of ‘the majority of criteria’ (or ‘the maximum group utility’). The value of v is usually taken as 0.5. However, it can take any value from 0 to 1 which define the strategy (Opricovic and Tzeng, 2004).

- 5 Rank the results according to S_i , R_i and Q_i in different cells. The best alternative is determined as the one with the maximum value. The i^{th} alternative with the maximum Q_i value is proposed as the compromise solution for given criteria weights, if conditions 1 and 2 are satisfied (Khorshidi and Hassani, 2013):

- *Condition 1*: ‘Acceptable advantage’ $Q(a'') - Q(a') \geq DQ$, where a'' is the alternative with the second-lower score in the ranking made according to Q_i and a' is the alternative with the lower score of Q_i .
- *Condition 2*: ‘Acceptable stability in decision making’: The alternative a' must have the maximum scores in the ranking made according to S_i and R_i . This compromise solution is stable within a decision-making process, which could be: ‘voting by majority rule’ (when $v > 0,5$ is needed) or ‘by consensus’ (when $v \approx 0.5$), or ‘with veto’ (when $v < 0,5$).

If one of the conditions is not fulfilled, one of the following alternatives can be adopted:

- a Alternatives a'' and a' if only *condition 2* is not satisfied.
- b Alternatives a', a'', \dots, a^m if *condition 1* not satisfied; a^m is determined by the relation $Q(a^m) - Q(a') \geq DQ$, for m as the maximum number of alternatives, the positions of these alternatives are ‘in closeness’.

2.8 EXPROM II method

Extended promethee II (EXPROM II) is a modification to promethee II, it treats ideal and non-ideal solutions. The great strength of this method was that it compares alternative by alternative using pairwise comparisons rather than criteria-criteria comparisons (Fadlina et al., 2017; Chatterjee et al., 2011). This particular characteristic allows a profound management of the whole data to finally obtain a more precise result. EXPROM II demands time and insight by part of the decision maker to avoid all possible mistakes along with the computation of the data. The data has to be previously standardised like Table 4, then the following steps can be computed (Fadlina et al., 2017).

- 1 Normalise the decision matrix with the following equation by considering the type of criteria among input and output.

- Output criteria are normalised by equation.

$$\hat{f}_{i,j} = \frac{f_{i,j} - \min_{h=1,\dots,m} f_{h,j}}{\max_{h=1,\dots,m} f_{h,j} - \min_{h=1,\dots,m} f_{h,j}} \quad ; \quad \begin{array}{l} i = 1, \dots, m \\ j = 1, \dots, k \end{array} \quad (35)$$

- Input criteria are normalised by equation.

$$\hat{f}_{i,j} = \frac{\max_{h=1,\dots,m} f_{h,j} - f_{i,j}}{\max_{h=1,\dots,m} f_{h,j} - \min_{h=1,\dots,m} f_{h,j}} \quad ; \quad \begin{array}{l} i = 1, \dots, m \\ j = k + 1, \dots, n \end{array} \quad (36)$$

Output criteria go from $j = 1$ representing the first to $j = k$ representing the last output criteria. Similarly, input criteria go from $j = k + 1$ representing the first to $j = n$ representing the last input criteria.

- 2 The number of pairs to compare can be calculated by using the following equation.

$$pairs = m(m - 1) \quad (37)$$

m is the number of alternatives and $pairs$ is the number of pairs to be created with the m alternatives for the next steps. For example, this work must compare 72 pairs due to there are 9 alternatives.

- 3 Calculate evaluative differences from alternative i^{th} with other alternatives by using the values of the matrix obtained in the normalisation, where $j = 1, \dots, n$ corresponds to the j^{th} criteria.

$$d_j(i, i') = \hat{f}_{i',j} - \hat{f}_{i,j} \quad ; \quad \begin{array}{l} j = 1, \dots, n \\ i \neq i' \end{array} \quad (38)$$

The matrix build by equation (38), will have $m(m - 1)$ rows and n columns.

- 4 Calculate preferences $P_j(i, i')$. There are six types of preferences functions, such as ordinary criterion, U-shape criteria, V-shape criteria, level criterion, V-shape with indifference criterion and Gaussian criterion, but the most common criteria used is the ordinary criterion as equation (39) shows.

$$P_j(i, i') = \begin{cases} 0, & \text{if } \dot{f}_{i,j} \leq \dot{f}_{i',j} \\ (\dot{f}_{i,j} - \dot{f}_{i',j}), & \text{if } \dot{f}_{i,j} > \dot{f}_{i',j} \end{cases} ; \quad \begin{matrix} j = 1, \dots, n \\ i \neq i' \end{matrix} \quad (39)$$

The matrix build by equation (39), will have $m(m-1)$ rows and n columns.

- 5 Calculate the weak preferences index with equation (40) which use the weight corresponding to the different criteria. The sum of all the weights is equal to 1.

$$WP(i, i') = \sum_{j=1}^n (P_j(i, i') \cdot w_j) \quad ; \quad i \neq i' \quad (40)$$

- 6 Define the strict preferences with equation (42) which are based in the evaluative distances $d_j(i, i')$ with the range of values as defined by the evaluation of the whole set of pairs.

$$L_j = 0 \quad ; \quad dm_j(i, i') = \max_{h=1, \dots, m} \dot{f}_{h,j} - \min_{h=1, \dots, m} \dot{f}_{h,j} \quad ; \quad j = 1, \dots, n \quad (41)$$

$$SP_j(i, i') = \frac{\max_{h=1, \dots, n} (0, d_h(i, i') - L_h)}{dm_j(i, i') - L_j} \quad ; \quad \begin{matrix} j = 1, \dots, n \\ i \neq i' \end{matrix} \quad (42)$$

L_j is the limit of preference which is equal to zero for this work in which ordinary criterion is used as preference function. The value of L_j is different from zero for different preference functions. The value of $dm_j(i, i')$ is equal to 1 for all the j^{th} criteria due that the matrix is normalised between 0 and 1. Thus, the maximum is 1 and the minimum is 0.

- 7 Compute the strict preference index with equation (43).

$$WSP(i, i') = \sum_{j=1}^n (SP_j(i, i') \cdot w_j) \quad ; \quad i \neq i' \quad (43)$$

- 8 Compute the value of total preference index with equation (44).

$$TP(i, i') = \min(1, WP(i, i') + WSP(i, i')) \quad ; \quad i \neq i' \quad (44)$$

- 9 Determine leaving and entering flow with equations (45) and (46).

- Leaving (positive) flow.

$$\varphi_i^+ = \frac{1}{m-1} \sum_{i'=1}^m TP(i, i') \quad ; \quad \begin{matrix} i = 1, \dots, m \\ i \neq i' \end{matrix} \quad (45)$$

- Entering (negative) flow.

$$\varphi_i^- = \frac{1}{m-1} \sum_{i'=1}^m TP(i', i) \quad ; \quad \begin{matrix} i = 1, \dots, m \\ i \neq i' \end{matrix} \quad (46)$$

m is the amount of alternatives that the decision matrix contains.

- 10 Calculate the net flow with equation (47).

$$\varphi_i = \varphi_i^+ - \varphi_i^- \quad ; \quad i = 1, \dots, m \quad (47)$$

- 11 The final ranking is determined in descending order by the the value of the net flow for each i^{th} alternative φ_i where the best choice has the higher φ_i value.

2.9 ORESTE method

It is a compensatory preference ranking approach. This method allows ranking the whole data of the decision matrix applying the Besson rank (BR). In many real-time situations, it is important to establish a link between qualitative and quantitative information. This method is particularly appropriate to support this type of conflicting decisions in absence of crisp numerical values and weights of the criteria (Pastijn and Leysen, 1989; Chatterjee et al., 2011). Oreste uses the qualitative information to determine the importance of each criterion, but in this work, AHP was applied previously to obtain quantitative data (weights) from qualitative information. Particularly, the weights calculated will be used as a replacement for subjective information to apply ORESTE method.

ORESTE method follows the next steps (Pastijn and Leysen, 1989):

- 1 Build the decision matrix as Table 4 example.
- 2 Determine the weak order of the criteria by using the weights to point the respective relevance.

C_x will be used as acronym for the j^{th} criteria where $x \in \mathbb{N} \rightarrow [1, n]$. The n criteria will be sorted in decreasing order, it means that the most important will be at the beginning of the left side the less important at the right side. The preference order is given by a preference structure with relations among j^{th} criteria determined by **I** or **P**, which represents preference and indifference respectively. Preference means that a criterion is preferred to another, while indifference means that two criteria have the same importance. To continue there is a little example about this step:

C₂PC₁PC₄IC₃

In this example, it is easy to observe that criteria 2 is most important than criteria 1, criteria 1 is most important than criteria 4 and criteria 3 is less important than the rest. Moreover, **P** means that criteria 2 has a higher weight (importance) than criteria 1 and **I** means that criteria 4 has the same weight (importance) than criteria 3.

- Determine for each criterion a weak order by sorting the alternatives similar to the previous step. A_x will be used as acronym for the i^{th} alternative where $x \in \mathbb{N} \rightarrow [1, m]$. To continue a few examples are shown applying the weak order rules.

$$\begin{aligned}
 C_1 : & \quad A_2 \mathbf{P} A_1 \mathbf{P} A_4 \mathbf{I} A_3 \dots A_m \\
 C_2 : & \quad A_4 \mathbf{I} A_2 \mathbf{P} A_3 \mathbf{I} A_1 \dots A_m \\
 & \quad \vdots \\
 C_n : & \quad A_1 \mathbf{I} A_3 \mathbf{P} A_2 \mathbf{P} A_4 \dots A_m
 \end{aligned}$$

The weak order is performed for each alternative and the meaning of **I** and **P** is the same. Where **I** is used for repeated scores and **P** for different values.

- BR is applied to: each criterion to rank each i^{th} alternative and the group of criteria. In order to rank each criterion individually for i^{th} alternatives, the type of criteria must be specified to distinguish among input and output. First, a common ranking $\in \mathbb{N} \Rightarrow [1, m]$ is computed; descending for output criteria and ascending for input criteria. The best alternative will be ranked by 1 and the worst with m , when two or more alternatives have the same score, the ranking order will not matter, but it is important to do not repeat the ranking numbers. Once common ranking finishes, BR appears to correct repeated rankings when there are two or more alternatives with the same ranking. In this case, the mean as equation (48) is performed by taking into account the repeated ranks of the alternatives and, finally the BR for them will be the mean value.

$$\bar{x} = \frac{1}{p} \sum_{h=1}^p x_h \tag{48}$$

p is the number of repeated ranks and x is the rank of the h^{th} alternative.

The group of criteria is ranking, in the same way, applying BR. First, a common ranking is performed for the criteria, then when there are two or more criteria with repeated weights, BR is applied. Finally, the mean value will be the rank for the repeated criteria weights.

Table 7 Decision matrix modified by BR

<i>Material</i>	C_1 type	C_2 type	C_3 type	\dots	C_n type
A_1	$bf_{1,1}$	$bf_{1,2}$	$bf_{1,3}$	\dots	$bf_{1,n}$
A_2	$bf_{2,1}$	$bf_{2,2}$	$bf_{2,3}$	\dots	$bf_{2,n}$
A_3	$bf_{3,1}$	$bf_{3,2}$	$bf_{3,3}$	\dots	$bf_{3,n}$
\vdots	\vdots	\vdots	\vdots	\ddots	\vdots
A_m	$bf_{m,1}$	$bf_{m,2}$	$bf_{m,3}$	\dots	$bf_{m,n}$
Weights	bw_1	bw_2	bw_3	\dots	bw_n

After to apply BR on the decision matrix, the final result has to be like Table 7, where bf is the BR for the alternatives of the j^{th} criteria and bw is the BR for the group of criteria.

- 5 Calculate the projection distances with equation (49). This projection allows complete compensation for the loss of one rank position of an action by an increase in the importance of the criterion by one rank position.

$$P_{i,j} = \frac{1}{2} [bf_{i,j} \cdot bw_j] \quad ; \quad \begin{array}{l} i = 1, \dots, m \\ j = 1, \dots, n \end{array} \quad (49)$$

$$\begin{bmatrix} P_{1,1} & P_{1,2} & \cdots & P_{1,n} \\ P_{2,1} & P_{2,2} & \cdots & P_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ P_{m,1} & P_{m,2} & \cdots & P_{m,n} \end{bmatrix} \quad ; \quad \begin{array}{l} i = 1, \dots, m \\ j = 1, \dots, n \end{array} \quad (50)$$

A matrix as (50) is obtained after applying projection distances equation.

- 6 Global BR for the matrix of projection distances (50). The ranking will be global without distinguishing among different criteria. First, a common ranking is performed in descending order, where the higher projection distance value will be rank as 1 and the lower as $(m \times n)$. When two or more items have the same projection distance value, the ranking order will not matter for them, but it is important to do not repeat the ranking numbers. Then, BR is finally applied similarly to step 4.

$$\begin{bmatrix} bP_{1,1} & bP_{1,2} & \cdots & bP_{1,n} \\ bP_{2,1} & bP_{2,2} & \cdots & bP_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ bP_{m,1} & bP_{m,2} & \cdots & bP_{m,n} \end{bmatrix} \quad ; \quad \begin{array}{l} i = 1, \dots, m \\ j = 1, \dots, n \end{array} \quad (51)$$

bP is the Besson ranking applied to the matrix of projection distances.

- 7 Aggregation: For each alternative, a summation is computed by applying equation (52).

$$r_i = \sum_{j=1}^n bP_{i,j} \quad ; \quad i = 1, \dots, m \quad (52)$$

- 8 The final ranking is computed in ascending order determined by the aggregation r value. The best alternative has the lower aggregation value and the worst has the higher value.

3 Results

The weight of each criterion has been computed by the AHP method regarding its importance for the geomembranes. After the determination of the weights of different

criteria using the AHP method with qualitative information, these weights were applied to the MCDM methods. The results have been developed with the methods COPRAS-G, OCRA, TOPSIS, VIKOR, EXPROM II, and ORESTE. The different steps involved in these methods were standardised to a single template and were discussed above. The results have been compared in order to determine their convergence and sensibility to finally ranked the best alternative.

3.1 *Criteria weighting*

The comparison among properties of every alternative are in Table I. The properties identification appears under the name of each property as P, C, WD, UVD, YM, SM, YS, TC, SH, and TE. The weight of each alternative was assigned according to the AHP method. The criteria weighting was firstly performed by the AHP method to obtain the subjective weights of different evaluation criteria. After the decision hierarchy for the problem was designed, the criteria were compared pairwise based on the experience of the author using the scale given in Subsection 2.1. In Table 8 can be showed the scale of relative importance used in the AHP method. The coefficients were assigned based on the characteristic for a geomembrane.

Table 8 Scale of relative importance

<i>Definition</i>	<i>Intensity of importance</i>
Equal importance	1
Moderate importance	3
Strong importance	5
Very strong importance	7
Extreme importance	9
Intermediate importance	2, 4, 6, 8

In Table 5 is illustrated the decision matrix generated for the geomembranes which take into account the importance of each criterion. The most important criteria to generate the matrix was considered (P); slightly more important were taken (C) and durability; strongly more important were considered mechanical properties; very strong importance were considered thermal properties. The results are consistent due to the value of the consistency index (CI = 0.018 for pipes and CI = 0.019 for the vessel). In Table 4 the weight coefficient of every criterion was determined. The most representative values are permeability (P) 46.9% and (C), 20.2%. On the other hand, less than 33% of the overall weight is distributed in WD, UVD, YM, SM, YS, TC, SH, and TE.

3.2 *COPRAS-G*

For the application of COPRAS-G method for the materials of the geomembranes, the related decision matrix is developed from the grey numbers applied in COPRAS-G are shown in Tables 2 and 3. Equations (5) and (6) allow to develop the normalised decision matrix which is then weighted normalised with equations (7) and (8), as is given in Tables 9 and 10. Sums of input and output criteria are shown in Table 11, which were calculated by using equations (9) and (10). Table 11 exhibits the priority

values Q_i and quantitative utility N_i values for the candidate alternatives of the pipes of Fischer-Tropsch reactor, as calculated using equations (12) and (13) respectively. Table 11 also shows the ranking of the alternative material as 7-4-5-6-2-3-1. PE-HD (general purpose, moulding and extrusion) and PP (homopolymer, low flow), obtain the first and second ranks respectively, in contrast PVC (flexible, Shore A60) alloy and ethylene propylene (diene) (EPDM/EPM, unreinforced) have the last rank.

Table 9 Weighted normalised decision matrix

Material	P		C		DW		DUV		YM	
1	0.2649	0.2924	0.0199	0.0242	0.0128	0.0171	0.0130	0.0195	0.0000	0.0000
2	0.0005	0.0014	0.0307	0.0362	0.0128	0.0171	0.0130	0.0195	0.0000	0.0000
3	0.0150	0.0158	0.0484	0.0570	0.0086	0.0128	0.0130	0.0195	0.0000	0.0001
4	0.1325	0.1633	0.0364	0.0401	0.0128	0.0171	0.0130	0.0195	0.0000	0.0000
5	0.0049	0.0083	0.0182	0.0200	0.0128	0.0171	0.0065	0.0130	0.0131	0.0155
6	0.0109	0.0182	0.0171	0.0192	0.0128	0.0171	0.0065	0.0130	0.0026	0.0050
7	0.0041	0.0058	0.0164	0.0181	0.0128	0.0171	0.0130	0.0195	0.0104	0.0106

Table 10 Continued of weighted normalised decision matrix

SM		YS		TC		SH		TE	
0.0000	0.0000	0.0022	0.0025	0.0013	0.0017	0.0018	0.0020	0.0017	0.0018
0.0000	0.0001	0.0027	0.0029	0.0017	0.0025	0.0020	0.0022	0.0017	0.0022
0.0000	0.0001	0.0045	0.0071	0.0017	0.0025	0.0020	0.0022	0.0017	0.0022
0.0000	0.0000	0.0003	0.0006	0.0013	0.0017	0.0022	0.0023	0.0022	0.0032
0.0018	0.0187	0.0073	0.0081	0.0017	0.0018	0.0018	0.0019	0.0011	0.0011
0.0032	0.0064	0.0022	0.0043	0.0013	0.0014	0.0020	0.0021	0.0032	0.0033
0.0134	0.0136	0.0058	0.0069	0.0038	0.0042	0.0019	0.0020	0.0011	0.0021

Table 11 Results of COPRAS-G method

P_i	R_i	Q_i	N_i	Ranking
0.0370	0.3025	0.0513	20.577%	7
0.0382	0.0364	0.1578	63.235%	4
0.0370	0.0701	0.0991	39.707%	5
0.0354	0.1888	0.0584	23.405%	6
0.0606	0.0268	0.2230	89.365%	2
0.0399	0.0359	0.1609	64.468%	3
0.0676	0.0239	0.2495	100.000%	1

3.3 OCRA

Firstly, standardised Table 5 were employed. The aggregate performance of each alternative with respect to all the output criteria is calculated with equations (16) and

(17). Input criteria are treated with equation (18), and subsequently, the linear preference ratings are calculated with equation (19). Finally, the overall preference rating for each alternative material is determined using equation (20). The detailed computations of this method for the geomembranes are illustrated in Table 12. In this method, the ranking material alternatives are obtained as 7-4-5-6-1-3-2, which suggests that PE-HD (general purpose, moulding and extrusion) and PP (homopolymer, low flow), obtain the first and second ranks respectively, in contrast PVC (flexible, Shore A60) alloy and ethylene propylene (diene) (EPDM/EPM, unreinforced) have the last rank.

Table 12 Results computed by OCRA method

i_i	I_i	o_i	O_i	$I_i + O_i$	P_i	Ranking
0.3765	0.0000	0.3638	0.2251	0.2251	0.0000	7
136.3136	135.9371	0.4866	0.3479	136.2850	136.0599	4
129.0072	128.6307	0.8064	0.6677	129.2984	129.0733	5
64.2618	63.8853	0.1387	0.0000	63.8853	63.6602	6
133.7363	133.3598	55.9952	55.8565	189.2163	188.9912	1
129.8001	129.4236	19.1058	18.9671	148.3907	148.1656	3
134.5418	134.1653	53.3695	53.2309	187.3962	187.1710	2

3.4 TOPSIS

The decision matrix is given in Table 5 was normalised using equation (21) for the application of the TOPSIS method and this was multiplied by the compromised weights obtained. Then the weighted normalised decision matrix was computed with equation (22) for the geomembranes. The ideal and nadir ideal solutions, determined by equations (23) and (24), are necessary to continue to the last steps for the pipes of the Fischer-Tropsch reactor. The distances from the ideal D_i^+ and non-ideal solutions D_i^- and the relative closeness to the ideal solution C_i were measured using equations (25), (26) and (27). The results for TOPSIS are presented in Table 13. The materials for the geomembranes could be ranked by the relative degree of approximation and the ranking is shown in Table 13. The ranking of the alternative material is 7-4-5-6-2-3-1. For TOPSIS method PE-HD (general purpose, moulding and extrusion) and PP (homopolymer, low flow), obtain the first and second ranks respectively, in contrast PVC (flexible, Shore A60) alloy and ethylene propylene (diene) (EPDM/EPM, unreinforced) have the last rank.

3.5 VIKOR

For VIKOR method, it is not necessary to normalise the decision matrix but the type of criteria is an important factor. Output and input criteria are treated with equations (28) and (29). Values for S_i , R_i and Q_i are calculated by equations (30), (31) and (34) respectively, whose values are shown in Table (14). Three different rankings are computed to corroborate that the best alternative is being chosen correctly. Condition 1 is accomplished as Table 15 shows. Condition 2 is also accomplished due that the worst alternative for S_i and R_i is the best for Q_i . In this method, the ranking material

alternatives is obtained as 7-3-5-6-2-4-1. It is revealed for VIKOR method PE-HD (general purpose, moulding and extrusion) and PP (homopolymer, low flow), obtain the first and second ranks respectively, in contrast PVC (flexible, Shore A60) alloy and ethylene propylene (diene) (EPDM/EPM, unreinforced) have the last rank.

Table 13 Results computed by TOPSIS method

D_i^+	D_i^-	C_i	Ranking
0.4138	0.0774	0.1576	7
0.0519	0.4162	0.8892	4
0.0949	0.3922	0.8052	5
0.2274	0.1982	0.4656	6
0.0376	0.4131	0.9166	2
0.0483	0.4008	0.8925	3
0.0200	0.4166	0.9543	1

Table 14 Results computed by VIKOR method

S_i	R_i	Q_i	Rankings		
			By S_i	By R_i	By Q_i
0.6563	0.4691	0.0000	1	1	7
0.1907	0.0918	0.8575	5	6	3
0.4128	0.2010	0.5329	3	3	5
0.5279	0.2481	0.3760	2	2	6
0.1536	0.1005	0.8794	6	4	2
0.2212	0.1005	0.8203	4	5	4
0.0843	0.0503	1.0000	7	7	1

Table 15 Condition 1 of VIKOR

Condition 1 is met	
a'	0.0000
a''	0.3760
$Q(a'') - Q(a')$	0.3760
DQ	0.1667
$Q(a'') - Q(a') \geq DQ$	

3.6 EXPROM II

EXPROM II is a powerful MCDM duet to realise a deep comparative analysis of alternative by alternative in pairs. As Table 16 shows, there are $m(m - 1)$ pairs of alternatives to be computed. First, it is important to normalise the decision matrix (Table 5) with equations (35) and (36) in order to obtain values between 0 and 1. It is indispensable to follow carefully the steps, Table 16 shows values obtained by equations (40), (43), (44), (45), (46) and (47). In this method, the ranking material

alternatives are obtained as 7-3-5-6-2-4-1. It is revealed for EXPROM II method PE-HD (general purpose, moulding and extrusion) and PP (homopolymer, low flow), obtain the first and second ranks respectively, in contrast PVC (flexible, Shore A60) alloy and ethylene propylene (diene) (EPDM/EPM, unreinforced) have the last rank.

Table 16 Results computed by EXPROM II method

<i>Pairs</i>	$WP(i, i')$	$WSP(i, i')$	$TP(i, i')$	φ_i^+	φ_i^-	φ_i	<i>Ranking</i>
1, 2	0.0661	0.0661	0.1323	0.1867	0.9103	-0.7236	7
1, 3	0.2759	0.2759	0.5518				
1, 4	0.1062	0.1062	0.2124				
1, 5	0.0508	0.0508	0.1016				
1, 6	0.0611	0.0611	0.1222				
1, 7	0.0000	0.0000	0.0000				
2, 1	0.5317	0.5317	1.0000	0.4653	0.1719	0.2934	3
2, 3	0.2342	0.2342	0.4683				
2, 4	0.3433	0.3433	0.6866				
2, 5	0.1200	0.1200	0.2399				
2, 6	0.1370	0.1370	0.2740				
2, 7	0.0616	0.0616	0.1231				
3, 1	0.5194	0.5194	1.0000	0.3684	0.5891	-0.2207	5
3, 2	0.0121	0.0121	0.0242				
3, 4	0.3036	0.3036	0.6073				
3, 5	0.1105	0.1105	0.2209				
3, 6	0.1242	0.1242	0.2483				
3, 7	0.0548	0.0548	0.1096				
4, 1	0.2346	0.2346	0.4691	0.1886	0.6715	-0.4829	6
4, 2	0.0061	0.0061	0.0121				
4, 3	0.1885	0.1885	0.3770				
4, 5	0.0645	0.0645	0.1290				
4, 6	0.0615	0.0615	0.1231				
4, 7	0.0106	0.0106	0.0212				
5, 1	0.5535	0.5535	1.0000	0.5209	0.1481	0.3727	2
5, 2	0.1571	0.1571	0.3141				
5, 3	0.3697	0.3697	0.7393				
5, 4	0.4388	0.4388	0.8776				
5, 6	0.0804	0.0804	0.1608				
5, 7	0.0166	0.0166	0.0333				
6, 1	0.4962	0.4962	0.9924	0.4344	0.2016	0.2328	4
6, 2	0.1065	0.1065	0.2129				
6, 3	0.3158	0.3158	0.6315				
6, 4	0.3683	0.3683	0.7365				
6, 5	0.0128	0.0128	0.0256				
6, 7	0.0036	0.0036	0.0072				
7, 1	0.5720	0.5720	1.0000	0.5773	0.0491	0.5282	1
7, 2	0.1680	0.1680	0.3359				
7, 3	0.3833	0.3833	0.7666				
7, 4	0.4542	0.4542	0.9085				
7, 5	0.0859	0.0859	0.1718				
7, 6	0.1405	0.1405	0.2810				

3.7 ORESTE

The tricky section of ORESTE method is to compute the BR, whose accuracy is directly related with the choice of the best alternative. Table 17 shows the BR applied to the alternatives of each j^{th} criteria. Steps 2 and 3 are optional because these do not affect to the final ranking but are useful to determine better the importance of each criterion and the alternatives of each criterion. Step 4 is applied in the decision matrix and the result is shown in Table 17. The projection distances are calculated with equation (49) whose result is ranked by BR. As step 6 explains, this BR is global and the result of this global ranking is shown in Table 18. With equation (52), the aggregation is calculated to finally rank in ascending order as Table 19 shows.

Table 17 BR for each criteria

Material	P 1	C 2	DW 3.5	DUV 3.5	YM 6	SM 6	YS 6	TC 9	SH 9	TE 9
1	7	4	3.5	4	5	6	6	5	6	3
2	1	5	3.5	1.5	6	5	5	2.5	2.5	4.5
3	5	7	7	1.5	4	4	3	2.5	2.5	4.5
4	6	6	3.5	4	7	7	7	6	1	6
5	3	3	3.5	6.5	1	2	1	4	7	1
6	4	2	3.5	6.5	3	3	4	7	4	7
7	2	1	3.5	4	2	1	2	1	5	2

Table 18 Global BR

Material	P	C	[DW	DUV	YM	SM	YS	TC	SH	TE
1	28	10.5	17	24	45.5	53.5	53.5	63.5	66	53.5
2	1	17	17	7.5	53.5	45.5	45.5	49.5	49.5	61.5
3	10.5	32.5	43	7.5	38.5	38.5	32.5	49.5	49.5	61.5
4	17	28	17	24	58	58	58	66	38.5	66
5	4.5	7.5	17	38.5	17	28	17	58	69	38.5
6	7.5	4.5	17	38.5	32.5	32.5	38.5	69	58	69
7	2.5	2.5	17	24	28	17	28	38.5	63.5	45.5

Table 19 Last results of ORESTE method

r_i	Ranking
415	6
347.5	3
363.5	4
430.5	7
295	2
367	5
266.5	1

4 Discussion

OCRA method was the unique method that ranked in second place to HDPE. This result is maybe occasioned due that there are few steps to follow and also the method does not require a normalisation of decision matrix as the previous step. Therefore, OCRA is not the best method to rank but is good to approximate the best alternative. In ORESTE method, steps 2 and 3 are not necessary steps. The theory behind these processes is tricky to understand however it can use to know the importance of each criterion and the relevance of the alternatives of each criterion. Currently, there are several codes for Matlab that calculate the ranking quickly but to understand each equation behind the process can open the mind to implement or develop better processes than existing.

Table 20 Global Rankings obtained from all methods

<i>Material</i>	<i>Rankings</i>					
	<i>OCRA</i>	<i>VIKOR</i>	<i>TOPSIS</i>	<i>COPRAS-G</i>	<i>EXPROM2</i>	<i>ORESTE</i>
PVC	7	7	7	7	7	6
CPE	4	3	4	4	3	3
CSPE	5	5	5	5	5	4
EPDM	6	6	6	6	6	7
PP	1	2	2	2	2	2
LLDPE	3	4	3	3	4	5
HDPE	2	1	1	1	1	1

Previous studies with HDPE in laboratory and field indicate that the projected service lives of HDPE geomembranes may range from many centuries to less than a decade depending on the material and exposure conditions (Rowe and Sangam, 2002). This information is supported with the MCDM methods whose result determines that HDPE is the best material for geomembranes.

5 Conclusions

MCDM methods are useful tools that expend qualitative and quantitative information to do the best choice. In this work, different types of materials for geomembranes were tested and the final result shows that the best choice is HDPE. This material obtained the best ranking in almost all the methods. On the other hand, PVC was ranking as the worst material for geomembranes in almost all. HDPE was chosen as the best material for geomembranes by using MCDM methods. These methods can be used instead of laboratory and field studies which take a lot of time and money. Therefore, mathematical models are useful for selecting the best materials for any application which uses qualitative and quantitative information.

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