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Application of material point method in modeling soil-geosynthetics interactions-a literature survey

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Abstract. The material point method (MPM) has garnered significant attention in recent years owing to its advantages in solving soil–water-structure interaction problems involving large deformations in geotechnical engineering. The MPM combines the benefits of point-based and mesh-based approaches (finite element method) with both Eulerian computational mesh and continuum descriptions of materials. The successful integration of MPM in simulated landslides, internal erosion, and excavation has been frequently reported. However, solving the soil–geosynthetic interaction problem with the MPM has not been explored, although such problems often entail large deformations. The goal of this study is to collate studies on the simulation of geosynthetics and their interactions with soil using MPM. This paper first discusses the basics of MPM and the formation of thin membrane materials using MPM. It also includes limited applications of MPM in simulating soil–geosynthetic interactions. The applications demonstrate that the MPM is particularly effective in resolving large deformation problems associated with geosynthetics, including problems of landfill settlement, reinforced-slope stability, and geocontainer dropping.

1. Introduction

Geosynthetics are extensively used in geotechnical engineering for various applications related to earth structures, including roadways, embankments, slopes, and retaining walls. The design considerations for these structures often encompass challenges, such as large total or localized deformations, instability, and bearing capacity failure. The integration of geosynthetics can mitigate these challenges; moreover, in certain cases, it enables the use of smaller quantities or low-quality construction materials [1,2].

In practice, geosynthetics are employed to closely interact with soil. Various studies have delved into the geosynthetic interactions, including experimental studies spanning from element tests, model tests, to field trials, in addition to numerical studies aiming to clearly depict these interactions. Widely used numerical methods for simulating soil–geosynthetic systems include the finite element method (FEM), finite difference method (FDM), and discrete element method (DEM).

Earth structures incorporating geosynthetics often face large deformation problems, such as the failure stages of reinforced slopes and retaining walls, failure of unpaved roads, and subsidence issues in landfills. Although FEM may be applied to these large deformation problems, it requires additional effort, such as mesh updates, to address mesh distortion issues. The DEM is proficient in resolving large deformation problems; however, it entails high computational costs. To overcome the limitations of the traditional FEM in large deformation problems with reasonable computational costs, the material point method (MPM) was introduced. The MPM is a hybrid method that combines a Lagrangian description of a continuum body with a Eulerian background mesh.

This study collates studies on the simulation of soil–geosynthetics interactions using the MPM. A brief overview of MPM is presented in Section 2.

2. Brief overview of MPM's history and algorithm

This section first outlines the history of MPM, followed by an overview of its algorithm, as exemplified by the one-phase single-point formulation of the standard MPM [3]. This formulation is frequently employed in scenarios involving dry or saturated soils where the excess pore pressure or solid–liquid relative movement is negligible. In cases where the effects of pore water cannot be ignored, alternative formulations, such as two-phase single-point, three-phase single-point, and two-phase double-point formulations, should be considered.

2.1. History of MPM

The MPM was first introduced in 1994 by Sulsky et al. [4] as an extension of the particle-in cell (PIC) method. The MPM is a mesh-free method developed to address the limitations of traditional FEM in simulating large-deformation problems.

The key innovation of MPM lies in its hybrid nature, which utilizes both Lagrangian and Eulerian descriptions. In the MPM, the continuum body is discretized into a set of Lagrangian particles, namely, material points, whereas the Eulerian mesh remains fixed in the background. These material points possess the material properties and move through a stationary background mesh, thereby enabling an accurate representation of the material deformation and flow.

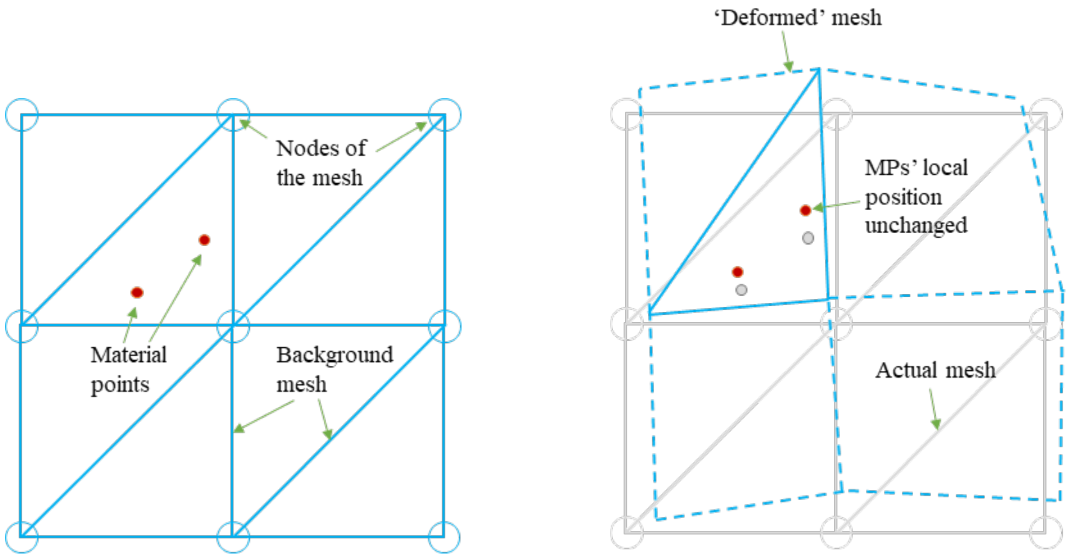
The advantages of MPM include avoiding the mesh entanglement issues that are commonly found in FEM and automatically implementing the no-slip and no-penetration contact algorithms because the MPM implements the velocity field when calculating the motion of particles. However, the MPM possesses some limitations such as computational costs, non-optimal positioning for numerical integration compared to FEM, and the grid-crossing problem, which refers to the stability issues caused by material points crossing mesh element boundaries. To provide a comprehensive overview of MPM, Vaucorbeila et al. [5] published a review covering over 300 references in 2020.

The MPM has been integrated to a spectrum of geotechnical problems, including landslide [6,7], tunnel stability analysis [8–10], pile driving [11–13], subsidence problems in landfills [14], and the installation of geocontainers [15]. Some applications offer solutions to soil–geosynthetic problems and are elaborated on in Section 3.

2.2. Algorithm of MPM (one-phase single-point)

As mentioned previously, MPM is a hybrid method that integrates both Lagrangian and Eulerian descriptions. Consequently, the computational loop for one time step in the MPM comprises two distinct phases: Lagrangian and convective (Eulerian).

2.2.1. Lagrangian phase. The Lagrangian phase in MPM closely resembles that in the FEM. The weak form of the virtual work equation is solved at the background mesh nodes. Subsequently, the nodal accelerations, velocities, and incremental displacements were determined based on these solutions, as shown in Figure 1. Notably, the background mesh itself remains stationary. The mesh deformation shown in Figure 1(b) is not a physical movement but a computational assignment of values to the mesh nodes. The material point values are then interpolated from these nodes using the local coordinate system of the mesh element within which each material point is situated.



(a) Initial positions of material points and mesh (b) Nodal values updated

Figure 1. Lagrangian phase of MPM.

For brevity, the weak form is omitted here, and the introduction begins with global matrices after integration. Expressed in terms of the momentum balance equation and posed at time t^k, the equation is

$$M_i{}^k \vec{a}_i{}^k = \vec{f_i}^{ext,k} - \vec{f_i}^{int,k} \tag{1}$$

where,

$M_i{}^k$: lumped mass matrix at time t^k

$\vec{a}_i{}^k$: nodal acceleration at time t^k

$\vec{f_i}^{ext,k}$: external force at time t^k

$\vec{f_i}^{int,k}$: internal force at time t^k

In this section, subscript *i* indicates the nodal values and subscript *p* indicates the particle values.

The nodal velocities in the MPM were computed using the nodal momentum, a modification introduced by Sulsky [16] in 1995 to the original MPM, aiming to circumvent division by nodal masses wherever possible. Dividing by a nodal mass close to zero can result in an unrealistically high velocity. Therefore, particle velocities (Equation (2)) were first updated to calculate the momentum (Equation (3)), and the nodal velocity was determined based on the momentum (Equation (4)).

$$\vec{v}_p{}^{k+1} = \vec{v}_p{}^k + \Delta t \sum_{i=1}^{nn,el} N_i(\vec{\xi}_p{}^k)\, \vec{a}_i{}^k \tag{2}$$

$$\vec{P}_i{}^{k+1} = \sum_{el=1}^{nel,i} \sum_{p=1}^{np} m_p N_i(\vec{\xi}_p{}^k)\, \vec{v}_p{}^{k+1} \tag{3}$$

$$\vec{v}_i{}^{k+1} = \frac{\vec{P}_i{}^{k+1}}{M_i{}^k} \tag{4}$$

where,

$\vec{v}_p{}^k, \vec{v}_p{}^{k+1}$: particle velocities at times t^k and t^{k+1}, respectively

Δt: Time step

$N_i(\vec{\xi}_p{}^k)$: shape function matrix

$\vec{P}_i{}^{k+1}$: nodal momentum at time t^{k+1}

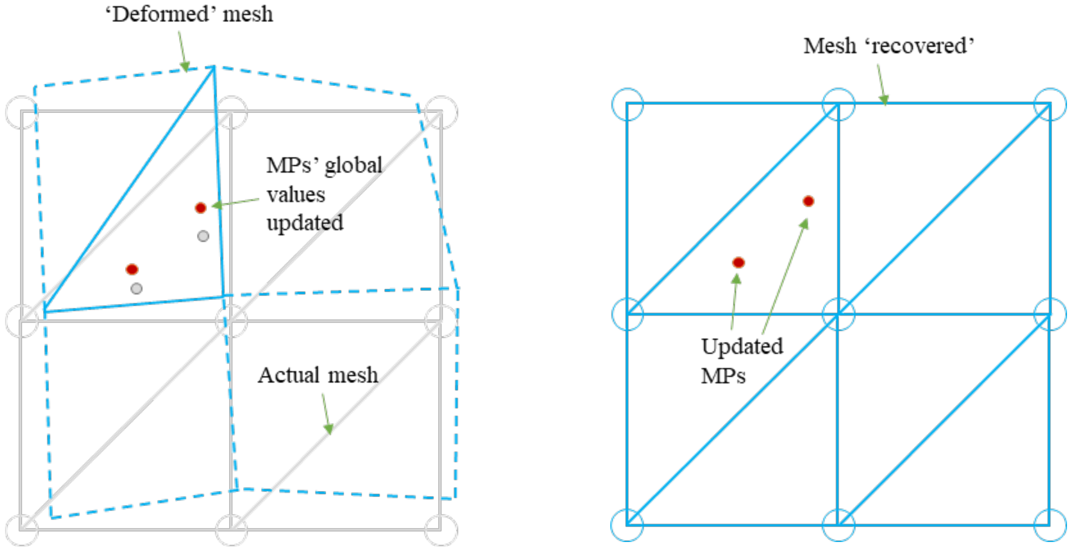
m_p: mass of particles

$\vec{v}_i^{\,k+1}$: nodal velocity at time t^{k+1}

The incremental nodal displacement $\Delta\vec{u}_i^{\,k+1}$ is determined by time step and nodal velocity (Equation (5)). Following this computation, the process transitioned to the convective phase.

$$\Delta\vec{u}_i^{\,k+1} = \Delta t\,\vec{v}_i^{\,k+1} \tag{5}$$

2.2.2 Convective phase. The convective phase primarily involves updating the material points using nodal data, erasing the nodal data, and initializing the mesh for the next time step, as shown in Figure 2. The movement of the material points occurs in this phase in a fixed background mesh, exemplifying the Eulerian characteristic of the MPM approach.



(a) Global values of material points updated (b) Mesh values erased

Figure 2. Convective phase of MPM.

The incremental nodal displacement $\Delta\vec{u}_i^{\,k+1}$ may help update the material point values; for example, updating the strain (Eq. 6), and displacement (Eq. 7).

$$\Delta\vec{\varepsilon}_p^{\,k+1} = B(\vec{\xi}_p^{\,k})\Delta\vec{u}_i^{\,k+1} \tag{6}$$

$$\vec{u}_p^{\,k+1} = \vec{u}_p^{\,k} + \sum_{i=1}^{nn,el} N_i(\vec{\xi}_p^{\,k})\,\Delta\vec{u}_i^{\,k+1} \tag{7}$$

where,

$B(\vec{\xi}_p^{\,k})$: the strain-displacement matrix

After all the material points are updated, the background mesh is restored to its original and undeformed states. This restoration erases all the nodal values, and the background mesh is initialized for the next time step.

3. Soil–geosynthetics interaction problems

This section synthesizes previous numerical studies on soil–geosynthetic interaction problems using various methods, including the MPM. Initially, the summary focused on research utilizing the FEM, FDM, and DEM. Subsequently, several studies based on the MPM are introduced. Owing to the relatively limited documentation of such simulation efforts, this review is confined to two illustrative examples: the interaction between soil and geomembranes in the context of landfill subsidence and the case of dropping geocontainers into the sea.

3.1. Soil–Geosynthetics interaction with other numerical methods

3.1.1 Composite approach. Several numerical methods aimed at simulating geosynthetics and interactions between soil and geosynthetics have been reported in existing literature. The simulation of the structure interactions dates back to the 1970s in studies of reinforced soil structures. The limited computational capacity at that time facilitated the introduction of a composite approach to model the soil-reinforcement system.

In 1976, Romstad et al. [17] proposed two types of analyses for reinforced soil structures—discrete and composite. Discrete analysis considers the components separately, enabling a direct simulation of their interactions. Although this method is practical, it is computationally expensive. In contrast, composite analysis utilizes an equivalent material matrix that relates the stress and strain of the composite, making it a more cost-effective option but without considering the slippage of the components. In the 1970s, several studies used this approach to simulate reinforced soil [18]. Over time, the composite approach has been mentioned less frequently owing to the growth of computational power. However, the composite approach is still used in certain studies, where it helps simplify and reduce the cost of modeling stabilized soil.

Chen et al. [19] employed a composite approach to simulate geocell-stabilized soil in the FDM analysis of geocell-stabilized retaining structures. A constitutive model for the composite material was adopted in accordance with the laboratory test results. The interface shear behavior was described separately according to the cable element in FLAC, including backfill soil-geocell stabilized soil interfaces and interfaces between geocell-stabilized layers. Song et al. [20] used ABAQUS, an FEM program, to develop a similar model for a geocell-stabilized retaining structure. In addition to the application of the composite approach to describe geocell-stabilized soils, surface-to-surface contact was assigned to the interfaces between the different materials where slip frictional contact was applied.

3.1.2 Discrete approach. Discrete modeling of geosynthetics and soils has garnered significant attention in recent years, in contrast to the composite approach, with FEM and FDM being frequently utilized owing to their user-friendly interfaces and computational efficiency. Commercial software, such as FLAC3D (FDM-based) and PLAXIS 3D (FEM-based), have developed specialized structural elements for geosynthetics, commonly referred to as Geogrid Elements [21,22]. These structural elements are designed as triangular elements capable of withstanding tensile forces alone, and they can effectively model the frictional contact between geosynthetics and soil. PLAXIS 3D's Geogrid elements typically require supplementary interface elements to model the frictional interactions between materials, whereas FLAC3D applies friction rules directly to the surface of the geogrid. Both the FLAC3D and PLAXIS 3D necessitate advanced settings to accommodate the geogrid's position and shape adjustments due to large deformations—FLAC3D employs a 'Large strain mode,' and PLAXIS 3D requires an 'updated mesh' for the geogrid elements.

The discrete element method (DEM) is another method suitable for modeling the interlocking behavior of geogrids and granular materials, driven by its innate capability to simulate frictional contacts. However, the high computational demand of the DEM is a significant drawback. To alleviate this, a combined FEM-DEM method is often employed, with the FEM representing the geogrid and the DEM particles depicting the soil. An interface or contact law at the contact surface is crucial for managing the frictional interactions and integrating two separate computational processes.

The Mohr–Coulomb criterion is the most prevalent contact model in the literature. This employs a linear function (Equation (8)), to discern the type of contact—either slip or stick—and to adjust the tangential stress as necessary. When the tangential stress does not exceed τ_{max}, the objects in contact maintain their position without slipping and the tangential stress remains unchanged. In contrast, if the tangential stress surpasses τ_{max}, a slip occurs, and the tangential stress is aligned with τ_{max}.

$$\tau_{max} = c + \sigma \cdot \tan\theta, \tag{8}$$

where, τ_{max} is the maximum shear stress at the interface, c is the intrinsic shear strength of the interface, σ is the normal stress applied on the interface, and $\tan\theta$ is the coefficient of friction of the interface.

The simplicity and efficacy of this criterion are rooted in its use of a linear function coupled with the assumption of static friction, while ignoring kinetic considerations. Despite overlooking the impact of the velocity and complex geometry of the contact interface, it proved to be a competent solution for the problems discussed in the literature.

A summary of the literature on the discrete simulation of the geosynthetic systems is presented in Table 1. A majority of the literature considers slip contact to decide whether to add interface elements at the contact face or apply contact conditions. The linear elastic material model for geosynthetics remains the most common choice, whereas Wang et al. [23] adopted a piecewise linear model for the geogrid, providing a more accurate description of the geogrid's tensile behavior.

Table 1. Summary of literature regarding geosynthetics-soil simulation.

Literature	Models	Method	Modelling of Geosynthetics	Contact type, method
Han et al. (2009) [24]	geocell-stabilized road base (gravel)	FLAC3D (FDM)	Geogrid element	Slip
Hegde and Sitharam (2015a, 2015b) [25,26]	geocell-stabilized road base (sand)	FLAC3D	Geogrid element	Slip
Ari and Misir (2021) [27]	Geocell-stabilized foundation	PLAXIS 3D (FEM)	Geogrid element, Updated Mesh	Slip, interface element
Demir et al. (2014) [28]	Geogrid-stabilized foundation	PLAXIS 3D	Plate element	No-slip
Han et al. (2012) [29]	Geogrid-reinforced embankment	PFC2D (DEM)	Bonded particles	Slip
Tran et al. (2014) [30]	Pullout test of geogrid-sand	FEM-DEM	Brick element	Slip, Interface element
Wang et al. (2014) [23]	Direct shear test of geogrid-sand	PFC2D	Bonded particles	Slip
Wang (2016) [31]	Pullout test of geogrid-granular	PFC2D	Bonded particles	Slip
Basudhar (2008) [32]	Geotextile in sand bed	FEM	Axial element	Slip, Contact conditions at the interface
Villard (2016) [33]	Geotextile under Granular embankments	FEM-DEM	Triangular element	Slip, contact law at the interface

The efficacies of the FEM and FDM may be compromised in scenarios involving large deformations. In such cases, the DEM is the preferred method of choice, especially for granular materials, as it excels in capturing their behavior with appropriate parameters. However, the computational demands of the DEM can constrain the scope of the model for practical handling. The MPM has emerged as a valuable addition in these contexts, adeptly handling the large deformations of continuous bodies and offering a computationally viable alternative for modeling extensive systems where DEM requirements are prohibitive.

3.2. Modeling of membrane in MPM

York et al. [34] proposed a formulation of thin membranes for 2D problems in MPM. A single layer of material points was employed throughout the membrane thickness, and a local coordinate system was assigned to each point to define the normal and tangential dimensions. The tangential dimension was delineated by a line connecting adjacent points, with the normal dimension oriented perpendicularly. The strain at each point was then projected onto this local coordinate system using stress calculations, as shown below. Notably, only the tangential-stress component was retained, discarding the rest to account for the inability of the membrane to withstand compressive loads.

Considering that many membrane materials are only stiff under tension and do not develop stress under compression, this study pragmatically set the compression side of the membrane stiffness to zero. This simplification, which does not consider buckling or bending, offers a cost-effective solution suitable for many engineering applications.

The inherent sticking contact algorithm of the MPM may result in nonphysical adhesion in scenarios where separation is expected, such as during impacts or frictional contacts. This issue stems from the single-valued velocity field defined by the linear element shape functions, which can inappropriately constrain bodies in close proximity within the same mesh element. To mitigate this, the authors proposed a criterion for grid nodes shared by distinct bodies, utilizing the normal component of the velocity to determine whether the bodies converged or diverged.

Gan et al. [35] proposed a membrane formulation for 3D problems in the MPM. In their approach, the membrane geometry was discretized into a mesh of triangular elements, with the vertices of these triangles constituting the material points of the membrane. The normal at each material point was calculated as the weighted average of the normals for all triangles sharing the vertex. This technique allows representation of the membrane surface within the MPM framework.

However, Lian et al. [36] highlighted a notable limitation regarding the precision of numerical integration within the MPM. They articulated that the material point quadrature method typically employed in MPM does not achieve the same level of accuracy as the Gaussian quadrature method used in FEM, particularly for membrane elements. They suggested that although MPM can model membranes in 3D, it may fall short of the accuracy attainable by FEM for such applications.

3.3. Composite approach in MPM simulation of fiber-reinforced soil

Guo et al. [37] developed an equivalent additional stress (EAS) approach to efficiently simulate fiber-reinforced soil using the MPM. The EAS approach considers the reinforcement effect of the fiber as a force acting on the soil by incorporating it into the constitutive law of soil.

The efficacy of the EAS method was demonstrated through simulations of triaxial compression and centrifuge model tests focusing on slope stability. The MPM simulations accurately reflected the large deformations observed in the triaxial test specimens, particularly beyond an axial strain of 10%. Furthermore, the model successfully depicted the progression of cracks during the slope tests, underscoring the potential of the EAS approach for capturing complex behaviors in fiber-reinforced soils.

3.4. Discrete approach in MPM simulation of soil-geomembrane interaction

Zhou et al. [14] adopted the quasi-static material point method (MPM) to model the complex interactions among geomembranes, soils, and waste in landfills, a scenario characterized by pronounced subsidence and consequential deformation. Their methodology introduced two significant enhancements to the standard MPM.

First, they implemented a partitioned MPM protocol to address the challenges associated with grid crossing effectively. This technique treats each material point as a discrete continuum entity that, upon navigation through the grid, it divides itself according to the grid cells in which it intersects. The innovation lies in the systematic mapping of the attributes of the material point, such as shape, volume, and density, to the intersected grid cells. The underlying assumption of this method is the initial uniform distribution of material points across all attributes, ensuring that the subdivisions of each material point remain unaffected by the deformation.

Second, an interface algorithm was integrated to emulate the geomembrane between the soil and landfill waste strata. The algorithm delineates the geomembrane by creating a unique tangential velocity field, which is different from that of the adjacent layers while aligning the normal velocity component. This arrangement enables the simulation of realistic frictional contact at the geomembrane interface. The process entails first pinpointing the grid elements containing geomembrane material points and then applying a friction-slip condition to these identified grid nodes. This condition dictates that if the tangential force exceeds the product of the normal force and friction factor, slippage occurs, necessitating an adjustment in the tangential force to reflect this. Due to MPM's absence of a direct force field at the material points, the model substitutes force with velocity components, employing the principle $F\Delta t = \Delta V m$ to translate forces into velocity changes.

3.5. Coupling of MPM with FEM in a simulation of geocontainers

Hamad et al. [15] utilized a combined FEM-MPM method to simulate the deployment of geocontainers —a type of geotextile bag filled with sand— in the sea. In their approach, the sand inside the geocontainer was modeled using the MPM, whereas the geotextile bag was discretized into a triangular FE mesh, wherein the vertices were material points in the MPM framework. The modelling of the geotextile is similar to that used by Gan et al. [35].

The stress and strain calculations of the geotextile bag were performed within the finite element framework by utilizing Gaussian integration at the Gaussian points of each triangular element. This calculation considered only the tensile forces adhering to the material behavior of the geotextile, thereby capturing its response under tension.

After stress calculation, the resultant membrane forces from the geotextile were integrated and transferred to the background mesh. These forces were subsequently incorporated into the global internal force calculations for the upcoming MPM time step. By applying FEM to the Gaussian quadrature of the geotextile, researchers enhanced the accuracy of the stress assessment and concurrently circumvented the grid-crossing issue, which is often encountered in MPM applications [3].

One limitation of the study was the assumption of non-slip contact between the sand and geotextile, which is the default condition in MPM simulations. This assumption does not account for the potential slippage at the interface, an aspect that may require further consideration for a more comprehensive simulation of geosynthetic–soil interactions.

4. Summary and discussion

This study synthesizes and analyzes state-of-the-art research on the numerical simulation of soil–geosynthetic interactions, with a particular focus on the use of MPM. The investigation also provides a detailed examination of the membrane formulation in MPM in addition to the geogrid structural elements in FEM and FDM. The key insights from this study are summarized as follows:

1. The versatility of the MPM is evident through its use in simulating large deformation scenarios in soil–geosynthetic systems. The limited studies in the literature have highlighted two specific implementations: a quasi-static MPM approach for addressing subsidence in landfills and an explicit dynamic MPM model for the deposition of geocontainers on the seabed. The choice of the simulation method must align with the characteristics and requirements of the problem.

2. Membrane formulations within MPM have shown considerable promise for tackling both two-dimensional and three-dimensional issues. This advancement sidesteps the mesh distortion challenges inherent in FEM and highlights the trade-off between the simplicity of MPM and the higher quadrature accuracy of Gaussian integration in FEM.

3. The MPM demonstrated capabilities in managing the membrane contact algorithm. Sticking contact is the default in the standard MPM, whereas slip friction contacts are often modeled by decoupling the tangential velocity at the interface and adjusting the frictional force to reflect slip conditions. A contact algorithm is applied to the background mesh nodes. This aspect of the MPM

framework may potentially lead to inaccuracies in the computation of contact forces or the application of contact conditions before the actual contact, particularly when the mesh size is relatively large in the vicinity of the contact interface. This facilitates the need for research on enhancing the precision of the contact algorithm within the MPM, particularly in the context of flexible materials such as geosynthetics.

4. Despite the successful applications mentioned here, simulations of soil–geosynthetic systems using MPM have not been extensively documented in the literature. The use of MPM to simulate large deformation problems involving geosynthetics holds significant potential, provided that a precise and dependable method for modeling soil–geosynthetic interactions is developed. This advancement could significantly enhance the effectiveness and accuracy of simulations in this field.

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