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Decision-making Under Certainty: a Linear Programming Approach towards Optimal Product Mix Decisions—a Case Study in Amhara Pipe Factory

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Abstract
Manufacturing organizations highly benefit from proper allocation of resources (working capital, raw materials, labor power, working times, machinery, etc.) to optimize a product mix which is useful for profit maximization. The main purpose of this study is to critically examine the products produced in Amhara Pipe Factory to certainly decide which of these products must be given more attention or produced more in order to maximize the profit. The products due considered are UPVC pipes, HDPE pipes and Geomembrane sheets. In the course of this study, linear programming technique will be used to make the best possible use of the total available productive resources of the factory (such as working time, raw material, machinery and equipment, and available labor power). Linear programming, an operations research technique is widely used in finding solutions to complex managerial decision problems, but Amhara Pipe Factory make more use of the trial-and-error method in making optimal product mix decisions. As such, the factory has been finding it difficult in allocating scarce resources in a manner that will ensure profit maximization. This study will be carried out to decide and arrive at the optimal product-mix of Amhara Pipe Factory. The production process of the firm will be formulated as a linear programming problem and the optimal product mix will be decided using POM-QM for Windows software.

Keywords: *Decision-making, Linear Programming, Amhara Pipe Factory, HDPE pipes*

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INTRODUCTION

The effectiveness of manufacturing industries, such as Amhara Pipe Factory, can grow better through good management decisions at the firm level which results in better output through either cost minimization or output maximization. This brings in increased production in the real sector. Thus, firm managers are always seeking for the right decisions so as to meet their objectives, which mainly revolves around how best to increase profit.

The aim of growth in manufacturing factories puts pressure on the management body in finding the way for optimal planning, organizing, leading and controlling levels of production [1]. As a result of this pressure, managerial decision-making tools for better performance of manufacturing factories are available to analyze business environments and to solve practical business problems such as deciding optimal product mix. Among these tools, is the use of quantitative techniques such as the linear programming model, which uses mathematical method in seeking the optimum course of action in any decision situation under the restriction of limited available resources.

One decision area for the management of any manufacturing firm is, determining the product mix of the firm. A product mix (also called product assortment), according to Philip Kotler [2], is the set of all product lines and items that a particular seller or producer offers for sale to buyers. Nowadays, manufacturing companies are facing with the general nature of scarcity of factors of production. As a result of the increasing cost of production, high customers' expectations and the general nature of the scarcity of factors of production, the management of manufacturing factories cannot be successful unless they take right actions for their production decisions. This enables them to achieve their profit-maximizing objective [3].

Amhara Pipe Factory produces twenty different types of products. But the factory uses the trial-and-error method in determining its product mix. This paper intends to establish that it is more profitable to use the linear programming technique in determining the product-mix of Amhara Pipe Factory by applying the linear programming techniques in determining the quantity combination of the different types of products (optimal product mix) with certainty.

OBJECTIVES OF THE STUDY

The main objectives of this study are:

- To formulate a linear programming model that would suggest a certain product-mix to ensure maximum revenue for Amhara Pipe Factory.
- To highlight the application of linear programming technique in determining the product-mix would be more profitable than the traditional trial and error method of producing.
- To attempt encouraging the company to adopt the application of linear programming technique in determining its product-mix despite some challenges of applying it in the production environment.

Statement of the Problem

The profitability of a manufacturing organization is highly tied to the volume of products sold which has direct relationship with the quality of the product, but Amhara Pipe Factory makes more use of trial and error method of determining the volume of products produced. As such, this firm has been finding it difficult in allocating scarce resources (raw materials, working hours, machinery, man power), in a manner that will ensure profit maximization or cost minimization.

The purpose of this study is to develop a linear programming model for the production process of Amhara Pipe Factory with all the necessary information and data known and determine an optimum product mix for one year.

LITERATURE REVIEW

The Decision-making Process

Decision-making is the action or process of making important decisions by identifying and choosing alternatives based on the available resources and tasks. It is simply the process of defining the problem, identifying the alternatives, determining the criteria, evaluating the alternatives, and choosing an alternative. Making a decision implies that there are alternative choices to be considered, and in such a case we not only have to identify as many of these alternatives as possible but to choose the one that best fits with our goals, objectives, desires, values, and so on [4].

Decision-making is an essential part of planning. Decision-making and problem solving are used in all management functions, although usually they are considered a part of the planning phase. Decision-making should start with the identification of the decision maker(s) and stakeholder(s) in the decision, reducing the possible disagreement about problem definition, requirements, goals and criteria. Then, a general decision-making process can be divided into the following steps [5]:

1. Identify and define the problem.
2. Listing of all possible future events, called states of nature, which can occur in the context of the decision problem.
3. Identification of all the courses of action (alternatives or decision choices) which are available to the decision-maker.
4. Expressing the payoffs resulting from each pair of course of action and state of nature. These payoffs are normally expressed in a monetary value.
5. Applying an appropriate mathematical decision analysis model to select best course of action from the given list.

Categories of Decision-making

Making decisions under certainty—Complete and accurate knowledge of outcome of each alternative should be there. There is only one outcome to each alternative.

- If the outcomes are known and the values of the outcomes are certain, the task of the decision maker is to compute the optimal alternative or outcome with some optimization criterion in mind.

- *As an example:* If the optimization criterion is least cost and you are considering two different brands of a product, which appear to be equal in value to you, one costing 20% less than the other, then, all other things being equal, you will choose the less expensive brand.
- However, decision-making under certainty is rare because all other things are rarely equal.
- Linear programming is one of the techniques for finding an optimal solution under certainty. Complex linear programming problems normally need computations with the help of a computer.

Making decisions under risk—Multiple possible outcomes of each alternative can be identified and a probability of occurrence can be attached to each.

- The making of decisions under risk, when only the probabilities of various outcomes are known, is similar to certainty.
- Instead of optimizing the outcomes, the general rule is to optimize the expected outcome.
- *As an example:* If you are faced with a choice between two actions, one offering a 1% probability of a gain of $10000 and the other a 50% probability of a gain of $400, you as a rational decision maker will choose the second alternative because it has the higher expected value of $200 as against $100 from the first alternative.

Making decisions under uncertainty—Multiple outcomes for each alternative can be identified but there is no knowledge of the probability to be attached to each.

- Decisions under uncertainty (outcomes known but not the probabilities) must be handled differently because, without probabilities, the optimization criteria cannot be applied.
- Some estimated probabilities are assigned to the outcomes and the decision-making is done as it is decision-making under risk.

Decision-making Under Certainty

Decision-making under *certainty* implies that we are certain of the future state of nature (or we assume that we are) in which all the functions are well defined. In this case, the decision-maker has the complete knowledge (perfect information) of consequence of every decision choice (course of action or alternative) with certainty. The *payoff table*, or *decision matrix*, shown in Table 1 will help in this discussion. A *payoff table* is a means of organizing a decision situation, presenting the payoffs from different decisions, given the various states of nature.

Table 1: *Payoff (Benefit) Table—Decision Matrix.*

Alternative	**State of nature/probability**			
	N_1	N_2	N_j	*Nn*
	p_1	p_2	p_j	*pn*
A_1	O_{11}	O_{12}	O_{1j}	O_{1n}
A_2	O_{21}	O_{22}	O_{2j}	O_{2n}
A_i	O_{i1}	O_{i2}	O_{ij}	O_{in}
A_m	O_{m1}	O_{m2}	O_{mj}	O_{mn}

Our decision will be made among some number *m* of alternatives, identified as A_1, A_2, ., A_m. The outcome of N may be more than one (the model allows for *n* different futures). These future states of nature may not be equally likely, but each state Nj will have some (known or unknown) probability of occurrence pj. Since, the future must take on one of the *n* values of Nj, the sum of the *n* values of pj must be 1.0.

The *outcome* (or payoff, or benefit gained) will depend on both the alternatives chosen and the future state of nature that occurs. For example, if we choose alternative Aj and state of nature Nj takes place (as it will with probability pj), the payoff will be outcome Oij. That means, there are *m* times *n* possible outcomes in the full payoff table.

Decision-making under *certainty* infers that we can make decisions certainly about the future state of nature. (In our model, this means that the probability p1 of future N1 is 1.0, and all other futures have zero probability) The solution, naturally, is to choose the alternative Aj that gives us the most favorable outcome Oij. Although this may seem like an insignificant problem, there are many complex problems that sophisticated and advanced mathematical techniques are needed to find the best solution.

Nearly all decisions are made in a situation of at least some uncertainty. But, the degree of uncertainty may vary from relative certainty to great uncertainty. Certain types of risks are integral part of decision-making practices. When we make decisions with certainty, it means we are rationally sure about what will happen when we make a decision. We do have information which we can rely on as well as the cause and effects of relationship are known.

Decision-making under certainty is a condition in which the decision maker has full and needed information to make a decision. Decision is made under the condition of certainty. The manager/the decision maker knows exactly what the outcome will be as she/he has enough clarity about the situation and knows the resources, time available for decision-making, the nature of the problem itself, possible alternatives to resolve the problem and undoubtedly clarify or certain with the results of alternatives. In most situations, the solutions are already available from the past experiences or incidents and are appropriate for the problem at hand. The decision to restock food supply, for example, when the goods in stock fall below a determined level is decision-making under circumstance of certainty [6].

Tools for Decision-making Under Certainty

Linear Programming—This is one of the common techniques for making decisions with certainty. In this method, a desired benefit (such as profit) can be expressed as a mathematical function (the value model or *objective function*) of several variables. The solution is the set of values for the independent variables (*decision variables*) that serves to maximize the benefit (or, in many problems, to minimize the cost), subject to certain limits (*constraints*). Linear programming techniques have found extensive applications in making a product mix decision; minimizing transportation costs, planning and scheduling production, and other areas.

The Analytic Hierarchy Process (AHP)—This method was developed by Thomas L. Saaty to solve problems with complex multi-criteria. AHP requires the decision maker to provide judgments about the relative importance of each criterion and then specify a preference for each decision alternative using each criterion. The yield of AHP is an ordered ranking of the decision alternatives based on the overall preferences expressed by the decision maker. The inclusion of subjective factors in coming to a recommended decision is allowed in this multi-criteria decision-making technique. It is a prominent tool for dealing with decisions under certainty, where subjective judgment is quantified in a logical manner and then used as a basis for reaching a decision. Ideas, feelings, and emotions affecting the decision process are measured and quantified to provide a numeric scale for prioritizing the alternatives in AHP. It is a structured technique for organizing and analyzing complex decisions based on mathematics and psychology. The general structure of AHP may include several hierarchies of criteria.

Algebra—For both certainty and uncertainty analyses, this basic mathematical logic is very beneficial. This method provides deterministic solutions such as break-even analysis and benefit cost analysis with valid and proper assumptions. Break-even analysis is an economic model which describes cost price-volume relationships. It is a complete certainty type of model because costs and revenues are known quantities. It is one of the techniques used to study the total cost, total revenue and output relationship. It indicates at what level of output, cost and revenue are in equilibrium. Therefore, it determines the level of operations in an organization where the operation neither gains a profit nor incurs a loss [7].

Linear Programming

According to Lieberman (2001) [8] and Suresh (2009) [9], linear programming was developed as a discipline in the 1940s, driven initially by the need to solve complex planning problems in war time operations. Linear programming developed rapidly in the post war periods as many manufacturing sectors found its valuable uses. It was founded by George B. Dantzig and John Von Neumann who created the simplex method in 1947 and who establish the theory of duality that same year, respectively. In 1975, mathematician Leonid Kantorovich (USSR) and economist Tjalling Koopmas (USA) were awarded the Nobel Prize for economics for their contribution to the theory of optimal allocation of resources, in which linear programming played a key role. Industries use linear programming as a tool, for example, for the purpose of allocating finite resources in an optimal way. Airline crew scheduling, shipping or telecommunication networks, oil refining and blending, stock and bond portfolio selection are some examples of application areas.

There are various opinions on the applicability of this technique to different decision-making practices developed over a long period of time following continuous improvement on the application of the technique in solving practical corporate problems. Most of the literature shows that the technique is a practical tool of analysis in allocating scarce resources to their optimal use and is of major reputation to the economies of poor countries. in an optimum manner so that the general productivity is maximized In an allocation problem, when there are a number of activities to be performed, alternative ways of doing them, and limited resources or facilities for performing each activity in the most possible effective way, the management is faced with the problem of how to best combine these activities and resources. This is known as optimization problem and can be approached using mathematical programming. Linear programming is also referred to as a uni-objective constrained optimization technique. This is because it seeks a single objective of either minimizing or maximizing unknown variables in a model. In line with this, linear programming deals with linear optimization of a function of variables known as objective function subject to set of linear equations and/or inequalities known as constraints. The constraints may be imposed by different resources such as market demand, production process and equipment storage capacity, raw material available, labor power, working hours and so on.

Linear programming, which is an operations research technique, is one of the best-known tools to management science. To sum up, the development of linear programming has been considered as one of the most important scientific advances of the past century, with a generally accepted practicality. Its impact since 1950 has been extraordinary. Today, it is the standard tool that has saved vast amount of money for many production companies [10].

METHODOLOGY

The general methodologies going to be used to accomplish the objectives of this study are interviews with operators, supervisors, shift leaders, and technical persons as well as randomly selected management members in order to gather the necessary data. Moreover, observation of work station, discussion and recording of past data are used.

Besides data collection and recording, the data collected for the products will be subjected to linear programming technique simplex method and will be solved using the POM-QM for Windows 3 software.

Generally, the following methods will be used during the course of research.

- Data collection through interview, observation and discussion
- Data recording for available raw material, machines, manpower and working hour
- Development of linear programming model for certain product mix

THE CASE STUDY: AMHARA PIPE FACTORY

Company Background

Amhara Pipe Factory PLC is the biggest among the very few factories that produce plastic products for different uses. The factory is specially supporting projects related with water sector development by saving time and foreign exchange (previously imported UPVC pipes, HDPE pipes as well as geomembrane sheets).

Amhara Pipe Factory is dedicated to produce customer demand driven UPVC, HDPE pipes and Geomembrane sheets that fulfill the national and international applicable quality requirements through the involvement of employees and stakeholders by implementing ISO 9001:2000 quality management system requirements and maintaining continual improvement of system performance via periodic internal audit and management review with an overriding aim to enhance customer satisfaction.

Amhara Pipe Factory has a vision of leading the pipe manufacturing industry by guaranteeing its clients all over the country as well as in East African region, the ultimate experience by upholding the factory's commitments and maintain the highest standards in manufacturing, quality, services, and trustworthy business practices. The factory's products are:

1. Unplasticized polyvinyl chloride (UPVC) pipes
2. High density polyethylene (HDPE) pipes
3. Geomembrane sheets

Amhara Pipe Factory

- ensures that the quality policy is communicated and understood by the work force
- develops smart quality objectives that are consistent with the quality policy and review periodically to ensure suitability with the fast changing national and global circumstances.

Amhara pipe factory has set the following quality objectives for the new year of 2018.

1. Earn sell revenue of 379, 083, 337. 00 birr and get gross profit of 49, 445, 665 birr.
2. To purchase 3403.169 tons of UPVC resin, 510.442 tons of coated calcium carbonate, 136.13 tons of stabilizer, 3.406 tons of titanium dioxide, 0.754 tons of carbon black, 1587.6 tons of PE 100 HDPE, 146.681 tons of black master batch and 2934.859 tons of LLDPE raw material which will keep the production in uninterrupted manner by considering possible machine down time and last year's performance.
3. To produce 3679.4 tons of quality UPVC pipes, 1540.00 tons of HDPE pipes and 2958.3 tons of geomembrane lining that fulfils customers' requirements.

The Factory and its Products

Amhara Pipe Factory (APF) is engaged in the production of different sizes, shapes, and lengths of plastic pipes known as UPVC and HDPE pipes, as well as geomembrane sheets. These products are differentiated by their sizes, thickness and lengths. The products of APF include the following as displayed in the Tables 2–4.

Table 2: *Various UPVC Pipe Products.*

UPVC Pipes		
S.N	**Outside Diameter, OD (mm)**	**Nominal Pressure, NP (Bar)**
1	50	6
2	75	4
3	110	4
4	160	10
5	200	16
6	280	16
7	355	16
8	400	10

Table 3: *Various HDPE Products.*

HDPE Pipes		
S.N.	*Outside Diameter, OD (mm)*	*Nominal Pressure, NP (Bar)*
9	16	25
10	50	10
11	75	10
12	90	10
13	110	10
14	160	10
15	200	16
16	250	16

Table 4: *Various geomembrane products.*

Geomembrane Sheets	
S.N.	*Thickness (mm)*
17	0.5
18	1.0
19	1.2
20	1.5

In order to produce these products, the firm requires different materials in different combinations. It requires machines of different types and sizes, skilled and unskilled labor, and raw materials. But for the purpose of this research work, we shall concentrate on raw materials, man hours, machine hours, and available machinery needed for production from the 1st day of Jan 2017 to the 31st day of Dec 2018. Other factors are held constant. The raw materials used by the factory for the production of the above products are:

- UPVC Resin (the major raw material)
- Coated Calcium Carbonate
- Titanium Dioxide
- Stabilizer
- Black Master Batch
- Carbon Black
- PE 100 HDPE
- LLDPE

During production, these raw materials are mixed in different proportions according to a pre-determined recipe of a product. Each raw material contributes a different percentage of the production cost.

Methodology Used for Case Study

Amhara Pipe Factory is chosen for this study for two main reasons. First, it uses the trial-and-error method in arriving at volume of products. Secondly, the factory produces twenty different products which makes the determination of the quantity combinations of the products produced (product mix) an important and major management decision. The research is designed to cover one year, 2015. This research will investigate the overall quantity combination of the twenty products produced by Amhara Pipe Factory during the research period and the allocation of resources to the various products. For this purpose, data were taken from records kept by the production supervisor and the marketing department relating to the different types of products produced by the factory; the production process, the available raw materials and their corresponding prices. I also conducted interviews with the manager on some issues that required his response.

The research then applies linear programming to determine a new quantity combination. The total contribution to revenue of each of the products for the year using the new quantity will now be compared with the total profit contribution made by the former product mix determined by the trial-and-error method. The problems encountered in the process will be noted and from personal interviews and relevant records, other peculiarities shall be established. When there are n choice variables and m constraints, the linear programming takes the general form with a linear objective function, a set of linear inequality constraints and a set of non-negativity restrictions as its major ingredients. The generalized n variable linear program can be stated as below:

Objective: Maximize $Z = C_1X_1 + C_2X_2 + \ldots\ldots + C_nX_n$ (objective function)

Subjected to Constraints:

$a_{11}x_1 + a_{12}x_2 + \ldots\ldots + a_{1n}x_n \leq r_1$

$a_{21}x_1 + a_{22}x_2 + \ldots\ldots + a_{2n}x_n \leq r_2$

$a_{m1}x_1 + a_{m2}x_2 + \ldots\ldots + a_{mn}x_n \leq r_m$

$x_j \geq 0$ $(j = 1, 2, \ldots, n)$ (non-negativity restrictions)

where, c_i, a_{ij} and r_i are given constants. The variables $x_1, x_2, \ldots\ldots, x_n$ are called decision variables. The problem is to find the values of the decision variables $(x_1, x_2, \ldots, x_n)$ which maximize the objective function Z, subject to the m constraints and the non-negativity restriction on the x_j variable. The output of the decision variables which maximize the objective function is called the optimal solution. This procedure for determining the optimal solution is called "Simplex Algorithm" and it will be solved by using POM software.

The Model

The model for use in this study, with the objective function of optimal product decision, is:

MAXIMIZE $Z = P_1X_1 + P_2X_2 + P_3X_3 + P_4X_4 + P_5X_5 + P_6X_6 + P_7X_7 + P_8X_8 + P_9X_9 + P_{10}X_{10} + P_{11}X_{11} + P_{12}X_{12} + P_{13}X_{13} + P_{14}X_{14} + P_{15}X_{15} + P_{16}X_{16} + P_{17}X_{17} + P_{18}X_{18} + P_{19}X_{19} + P_{20}X_{20}$

SUBJECTED TO THE FOLLOWING CONSTRAINTS:

$C_{11}X_1 + C_{12}X_2 + C_{13}X_3 + C_{14}X_4 + C_{15}X_5 + C_{16}X_6 + C_{17}X_7 + C_{18}X_8 + \ldots\ldots C_{30}X_{20} \leq B_1$

$C_{21}X_1 + C_{22}X_2 + C_{23}X_3 + C_{24}X_4 + C_{25}X_5 + C_{26}X_6 + C_{27}X_7 + C_{28}X_8 + \ldots\ldots C_{40}X_{20} \leq B_2$

$C_{31}X_1 + C_{32}X_2 + C_{33}X_3 + C_{34}X_4 + C_{35}X_5 + C_{36}X_6 + C_{37}X_7 + C_{38}X_8 + \ldots\ldots C_{50}X_{20} \leq B_3$

$$C_{41}X_1 + C_{42}X_2 + C_{43}X_3 + C_{44}X_4 + C_{45}X_5 + C_{46}X_6 + C_{47}X_7 + C_{48}X_8 + \ldots\ldots\ldots\ldots C_{60}X_{20} \leq B_4$$

$$C_{51}X_1 + C_{52}X_2 + C_{53}X_3 + C_{54}X_4 + C_{55}X_5 + C_{56}X_6 + C_{57}X_7 + C_{58}X_8 + \ldots\ldots\ldots\ldots C_{70}X_{20} \leq B_5$$

$$C_{61}X_1 + C_{62}X_2 + C_{63}X_3 + C_{64}X_4 + C_{65}X_5 + C_{66}X_6 + C_{67}X_7 + C_{68}X_8 + \ldots\ldots\ldots\ldots C_{80}X_{20} \leq B_6$$

$$C_{71}X_1 + C_{72}X_2 + C_{73}X_3 + C_{74}X_4 + C_{75}X_5 + C_{76}X_6 + C_{77}X_7 + C_{78}X_8 + \ldots\ldots\ldots\ldots C_{90}X_{20} \leq B_7$$

$$C_{111}X_1 + C_{112}X_2 + C_{113}X_3 + C_{114}X_4 + C_{115}X_5 + C_{116}X_6 + C_{117}X_7 + C_{118}X_8 + \ldots C_{130}X_{20} \leq B_{20}$$

$$X_i \geq 0$$

where, Z = total revenue of the various products of APF for the year of 2015.
$P_{1\ldots.8}$ = selling prices coefficients
$X_{1\ldots8}$ = the various products produced by APF.
C = the numerical values that expresses the per unit usage of the available resource.
$B_{1\ldots.8}$ = the resource values that we seek to fully utilize.
The optimal values of the different types of products produced by the factory will show the combination (product mix) obtained through the application of linear programming model. The current selling price of each product is shown in Table 5.

Facts and Assumptions

- The production quantity is same as the sales. As it is unrealistic to produce what is not sold.
- There is a linear relationship among the variables used in the model.
- There is a continuous supply of electricity power.
- There are 353 net working days in a year.
- There are 16 neat working hours in a day which we get by subtracting machine cleaning hours, waiting hours for required temperatures, preparing chemicals, and so on from the 24 hours.
- The available machines run with an efficiency of 80%.

The factory's available resources are shown in Table 6. The recipe to produce various products is shown in Table 7. Weight per meter of the various products is shown in Table 8.

Table 5: *Unit selling prices of the products.*

S.N.	Products	Unit selling price (Birr/kg)
1	50 mm UPVC pressure pipes	99.35
2	75 mm UPVC pressure pipes	169.00
3	110 mm UPVC pressure pipes	305.70
4	160 mm UPVC pressure pipes	1133.05
5	200 mm UPVC pressure pipes	1581.41
6	280 mm UPVC pressure pipes	2029.80
7	355 mm UPVC pressure pipes	5292.77
8	400 mm UPVC pressure pipes	6880.90
9	16 mm HDPE pressure pipes	6.50
10	50 mm HDPE pressure pipes	28.60
11	75 mm HDPE pressure pipes	131.00
12	90 mm HDPE pressure pipes	92.50
13	110 mm HDPE pressure pipes	116.70
14	160 mm HDPE pressure pipes	193.50
15	200 mm HDPE pressure pipes	656.25
16	250 mm HDPE pressure pipes	1019.95
17	0.5 mm thick geomembrane sheets	33.65
18	1.0 mm thick geomembrane sheets	58.65
19	1.2 mm thick geomembrane sheets	80.30
20	2.0 mm thick geomembrane sheets	133.50

Table 6: *Available resources.*

Available Raw Materials			**Available Machines**	**Machine Hours**	**Man Hours**
Product Type	Name of Raw Material	Amount available for the year (tons)			
UPVC Pipes	PVC	3403.169	Four	353*16*60*0.8*4 =1084416 min	Insignificant
	$CaCO_3$	510.442			
	Stabilizer	136.13			
	TiO_2	3.406			
	Carbon Black	0.754			
HDPE Pipes	PE 100 HDPE	1587.6	Two	353*16*60*0.8*2 =542208 min	Insignificant
Geo-membrane sheets	FB 2310	1467.45	One	353*16*60*0.8*1 =271104 min	Insignificant
	FB 2230	1467.45			
	Black Master Batch	146.681			

Table 7: *Recipe for the various products.*

S.N.	**Products**	**Consumption per Kg of product (Kg)**								
		UPVC Resin	*$CaCO_3$*	*Stabilizer*	*TiO_2*	*CB*	*PE100 HDPE*	*FB 2310*	*FB 2230*	*BMB*
1	50 mm UPVC pipes	1.40	1.15	1.1	1.01	1.01	0	0	0	0
2	75 mm UPVC pipes	1.65	1.17	1.14	1.02	1.02	0	0	0	0
3	110 mm UPVC pipes	1.84	1.19	1.17	1.04	1.06	0	0	0	0
4	160 mm UPVC pipes	1.96	1.2	1.21	1.08	1.11	0	0	0	0
5	200 mm UPVC pipes	2.64	2.21	2.27	2.11	2.16	0	0	0	0
6	280 mm UPVC pipes	2.87	2.24	2.34	2.17	2.21	0	0	0	0
7	355 mm UPVC pipes	3.88	3.30	3.40	3.23	3.26	0	0	0	0
8	400 mm UPVC pipes	4.47	3.80	3.88	3.39	3.46	0	0	0	0
9	16 mm HDPE pipes	0	0	0	0	0	1	0	0	0
10	50 mm HDPE pipes	0	0	0	0	0	1.72	0	0	0
11	75 mm HDPE pipes	0	0	0	0	0	2.66	0	0	0
12	90 mm HDPE pipes	0	0	0	0	0	3.78	0	0	0
13	110 mm HDPE pipes	0	0	0	0	0	4.55	0	0	0
14	160 mm HDPE pipes	0	0	0	0	0	5.4	0	0	0
15	200 mm HDPE pipes	0	0	0	0	0	6.8	0	0	0
16	250 mm HDPE pipes	0	0	0	0	0	8.4	0	0	0
17	0.5 mm thick geomembrane sheets	0	0	0	0	0	0	1.40	1.1	1.05
18	1.0 mm thick geomembrane sheets	0	0	0	0	0	0	2.80	2.2	2.10
19	1.2 mm thick geomembrane sheets	0	0	0	0	0	0	3.70	2.76	2.53
20	2.0 mm thick geomembrane sheets	0	0	0	0	0	0	5.5	3.4	3.2

Note - HDPE pipes use only PE 100 raw material.

Table 8: *Weight per meter of the various products.*

S.N.	**Products**	**Average speed of machine**	**Weight per meter (Kg/meter)**	**Time to process 1kg of product (min)**
1	50 mm UPVC pressure pipes	5 meter/min	0.34	0.2
2	75 mm UPVC pressure pipes	5 meter/min	0.7	0.2
3	110 mm UPVC pressure pipes	5 meter/min	1	0.2
4	160 mm UPVC pressure pipes	5 meter/min	4.39	0.2
5	200 mm UPVC pressure pipes	5 meter/min	9.84	0.2
6	280 mm UPVC pressure pipes	5 meter/min	21.88	0.2
7	355 mm UPVC pressure pipes	5 meter/min	34.5	0.2
8	400 mm UPVC pressure pipes	5 meter/min	40	0.2
9	16 mm HDPE pressure pipes	5 meter/min	0.24	0.2
10	50 mm HDPE pressure pipes	5 meter/min	0.65	0.2
11	75 mm HDPE pressure pipes	5 meter/min	1.03	0.2
12	90 mm HDPE pressure pipes	5 meter/min	1.45	0.2
13	110 mm HDPE pressure pipes	5 meter/min	1.78	0.2
14	160 mm HDPE pressure pipes	5 meter/min	4.6	0.2
15	200 mm HDPE pressure pipes	5 meter/min	6.5	0.2
16	250 mm HDPE pressure pipes	5 meter/min	7.8	0.2
17	0.5 mm thick geomembrane sheets	10 m/min	0.5	0.1
18	1.0 mm thick geomembrane sheets	10 m/min	1.16	0.1
19	1.2 mm thick geomembrane sheets	10 m/min	1.2	0.1
20	2.0 mm thick geomembrane sheets	10 m/min	2.3	0.1

Based on all the information provided, APF can be translated into the model below:

$$\text{MAXIMIZE } Z = 99.35X_1 + 169X_2 + 305.70X_3 + 1133.05X_4 + 1581.41X_5 + 2029.80X_6 + 5292.77X_7 + 6880.90X_8 + 6.50X_9 + 28.60X_{10} + 131.00X_{11} + 92.50X_{12} + 116.70X_{13} + 193X_{14} + 656.25X_{15} + 1019.95X_{16} + 33.65X_{17} + 58.65X_{18} + 80.30X_{19} + 133.50X_{20}$$

SUBJECT TO CONSTRAINTS:

$1.4X_1 + 1.65X_2 + 1.84X_3 + 1.96X_4 + 2.64X_5 + 2.87X_6 + 3.88X_7 + 4.47X_8 \leq 3403169$

$1.15X_1 + 1.17X_2 + 1.19X_3 + 1.20X_4 + 1.21X_5 + 1.24X_6 + 3.30X_7 + 3.80X_8 \leq 510442$

$1.1X_1 + 1.14X_2 + 1.17X_3 + 1.21X_4 + 2.27X_5 + 2.34X_6 + 3.40X_7 + 3.88X_8 \leq 136130$

$1.01X_1 + 1.02X_2 + 1.04X_3 + 1.08X_4 + 2.11X_5 + 2.17X_6 + 3.23X_7 + 3.39X_8 \leq 3406$

$1.01X_1 + 1.02X_2 + 1.06X_3 + 1.11X_4 + 2.16X_5 + 2.21X_6 + 3.26X_7 + 3.46X_8 \leq 754$

$1X_9 + 1.72X_{10} + 2.66X_{11} + 3.78X_{12} + 4.550X_{13} + 5.40X_{14} + 6.80X_{15} + 8.4X_{16} \leq 1587600$

$1.40X_{17} + 2.80X_{18} + 3.70X_{19} + 5.5X_{20} \leq 1467450$

$1.10X_{17} + 2.20X_{18} + 2.76X_{19} + 3.4X_{20} \leq 1467450$

$1.05X_{17} + 2.1X_{18} + 2.53X_{19} + 3.2X_{20} \leq 146681$

$0.2X_1 + 0.2X_2 + 0.2X_3 + 0.2X_4 + 0.2X_5 + 0.2X_6 + 0.2X_7 + 0.2X_8 \leq 1084416$

$0.2X_9 + 0.2X_{10} + 0.2X_{11} + 0.2X_{12} + 0.2X_{13} + 0.2X_{14} + 0.2X_{15} + 0.2X_{16} \leq 542208$

$0.1X_{17} + 0.1X_{18} + 0.1X_{19} + 0.1X_{20} \leq 271104$

$X_1, X_2 \ldots\ldots\ldots\ldots\ldots\ldots, X_{20} \geq 0$

where, the decision variables are:

X_1 = The quantity of 50 mm UPVC pressure pipes to be produced.
X_2 = The quantity of 75 mm UPVC pressure pipes to be produced.
X_3 = The quantity of 110 mm UPVC pressure pipes to be produced.
X_4 = The quantity of 160 mm UPVC pressure pipes to be produced.
X_5 = The quantity of 200 mm UPVC pressure pipes to be produced.
X_6 = The quantity of 280 mm UPVC pressure pipes to be produced.
X_7 = The quantity of 355 mm UPVC pressure pipes to be produced.
X_8 = The quantity of 400 mm UPVC pressure pipes to be produced.
X_9 = The quantity of 16 mm HDPE pressure pipes to be produced.
X_{10} = The quantity of 50 mm HDPE pressure pipes to be produced.
X_{11} = The quantity of 75 mm HDPE pressure pipes to be produced.
X_{12} = The quantity of 90 mm HDPE pressure pipes to be produced.
X_{13} = The quantity of 110 mm HDPE pressure pipes to be produced.
X_{14} = The quantity of 160 mm HDPE pressure pipes to be produced.
X_{15} = The quantity of 200 mm HDPE pressure pipes to be produced.
X_{16} = The quantity of 250 mm HDPE pressure pipes to be produced.
X_{17} = The quantity of 0.5 mm thick geomembrane sheets to be produced.
X_{18} = The quantity of 1.0 mm thick geomembrane sheets to be produced.
X_{19} = The quantity of 1.2 mm thick geomembrane sheets to be produced.
X_{20} = The quantity of 2.0 mm thick geomembrane sheets to be produced.

Data Analysis, Results, and Discussion

The data were analyzed using the POM-QM for Windows 3 software, and the solutions are given in Table 9.

When we investigate the factory's current revenue with possible revenue, it could get its used linear programming as shown in Table 10.

In Table 10, the optimal solution of the problem is shown for products of APF. The production level of 400 mm UPVC pipes, 250 mm HDPE pressure pipes and 2 mm thick geomembrane sheets yielded 8720 kg, 1,474,200 kg and 105421.4 kg respectively while the objective function yielded 1,577,685,495 Birr. Application of linear programming for decision-making indicates that the company should produce only the above amounts of products certainly to get the maximum revenue.

***Table 9:** Certain units of products to be produced.*

S.N.	Products	Values to be produced with certainty	Total
1	50 mm UPVC pressure pipes	0	218 meters = 218*40 =8720 kg
2	75 mm UPVC pressure pipes	0	
3	110 mm UPVC pressure pipes	0	
4	160 mm UPVC pressure pipes	0	
5	200 mm UPVC pressure pipes	0	
6	280 mm UPVC pressure pipes	0	
7	355 mm UPVC pressure pipes	0	
8	400 mm UPVC pressure pipes	218m	
9	16 mm HDPE pressure pipes	0	189000meters =189000*7.8 =1474200 kg
10	50 mm HDPE pressure pipes	0	
11	75 mm HDPE pressure pipes	0	
12	90 mm HDPE pressure pipes	0	
11	110 mm HDPE pressure pipes	0	
14	160 mm HDPE pressure pipes	0	
15	200 mm HDPE pressure pipes	0	
16	250 mm HDPE pressure pipes	189000m	
17	0.5 mm thick geomembrane sheets	0	45838 meters =45838*2.3 =105427.4 kg
18	1.0 mm thick geomembrane sheets	0	
19	1.2 mm thick geomembrane sheets	0	
20	2.0 mm thick geomembrane sheets	45838m	

***Table 10:** Comparison of decision-making with trial-and-error vs decision-making with certainty.*

	Total amount of products to be produced			
	UPVC products (tons)	*HDPE products (tons)*	*Geomembrane sheets (tons)*	*Total revenue (Birr)*
Management decision-making (trial and error method)	3679.4	1540.0	2958.3	379,083,337
Decision-making with certainty using LP	8720 kg	1474200 kg	105421.4 kg	1, 577, 685, 495

CONCLUSION AND RECOMMENDATIONS

Based on the data collected from Amhara Pipe Factory, the study has successfully decided the product mix of Amhara Pipe Factory using linear programming decision-making with certainty approach. In the process, the certain quantities of the various pipes to be produced with all the available resources known within the study period were established. Also, the status of the resources and the unit selling prices of each product to the objective function were known. This is the advantage of using the linear programming decision-making with certainty tool.

Another issue becomes, how the management (decision-making) body of APF could be exposed to the complex steps involved in arriving at the optimal values of products, using the linear programming model for decision-making. From the researchers' personal observations in the course of this study, APF has no employees skilled in the operations research techniques who also possess a broad understanding of business environment and knowledge of the managerial roles and functions. As such, the firm should seek outside consultants and advisors to bring this and/or other techniques to bring effect on management's decision-making problems. This can help to assist the decision-making body of the factory for short period of time.

However, for the future, APF should gain well trained and experienced employees who can make use of this new technique effectively, efficiently and interpret the results of the analysis to top managers in the factory's overall context. Moreover**,** the decision-making process and the implementation will be easier if the decision makers and the all respective bodies can work together to achieve the best outcome with respect to degree of certainty.

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REFERENCES

1. Taylor BW. *Introduction to Management Science.* Eleventh Ed. Prentice Hall; Englewood Cliffs, NJ, 2013.
2. Philip Kotler. *Marketing Decision Making: A Model Building Approach*. New York: Harper and Row Publishers; 1983.
3. Fagoyinbo IS. Maximization of profit in manufacturing industries using linear programming techniques. *Mediterranean Journal of Social Sciences*. 2011; 2(6): 97–105.
4. Harris R. (1998). *Introduction to Decision Making part 1.* VirtualSalt: Version Date: June 9, 2012 http://www.virtualsalt.com/crebook5.htm [access Dec 2019].
5. Anderson David R, Sweeney DJ. *An Introduction to Management Science: Quantitative Approaches to Decision Making.* USA: South-Western Cengage Learning; 2010.
6. Kareem B, Aderoba AA. Linear programming based effective maintenance and manpower planning strategy. *International Journal of the Computer, the Internet and Management*. 2008; 16(2): 26–34.
7. Taha HA. *Operations Research: An introduction.* India: Pearson Education; 2013.
8. Lieberman HF. *Introduction to Operations Research.* New Delhi: McGraw-Hill; 2012.
9. Suresh SA. *Operations Management.* New Delhi: New Age International; 2009.
10. Amakom BI. Optimizing profit with the linear programming model. *Interdisciplinary Journal of Research in Business*. 2012; 2(2): 37–49.

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